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NUMERICAL ASSESSMENT OF THE COMPUTER CODES FOR ANALYZING BOUNDA--ETC(U)

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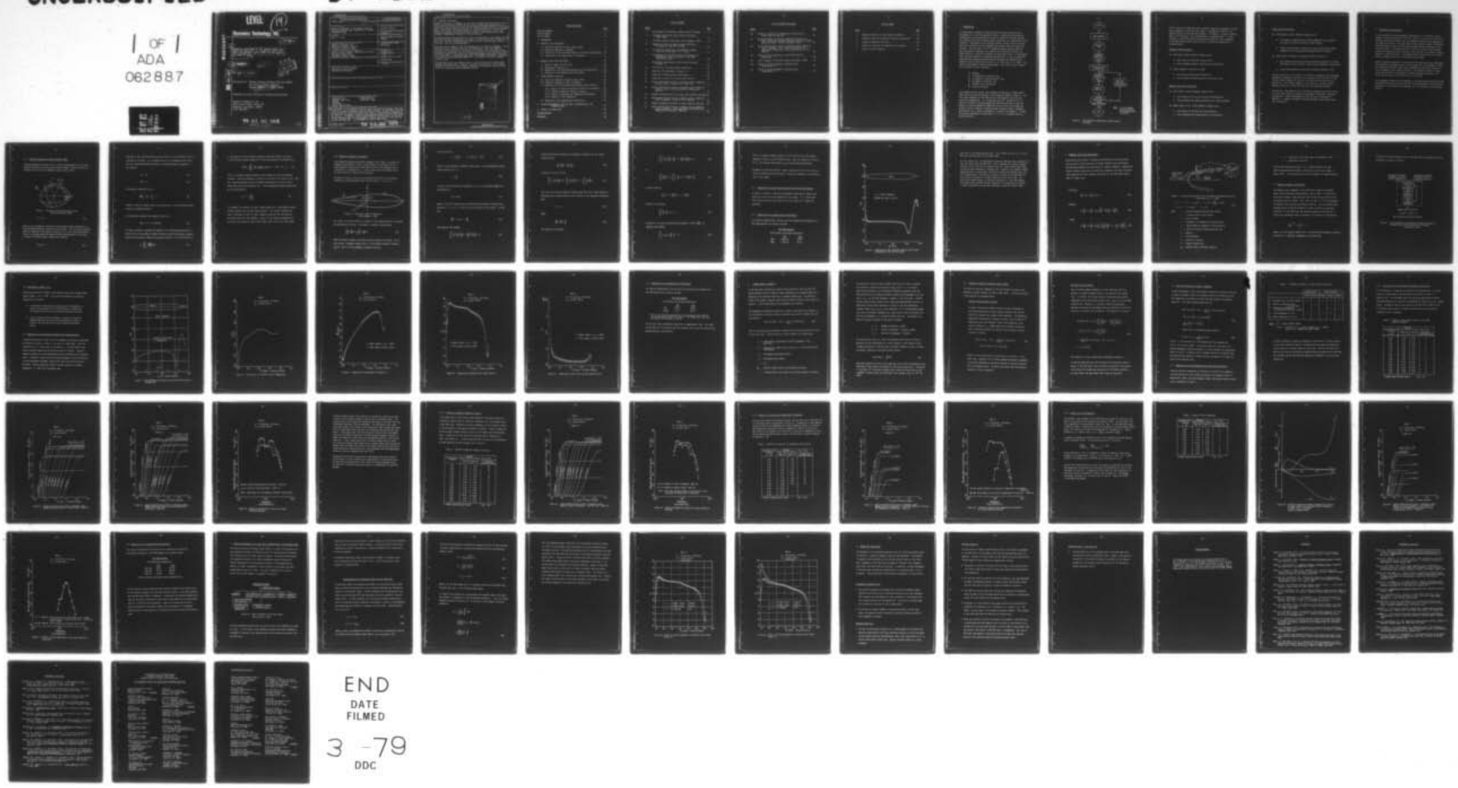

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# Dynamics Technology, Inc.

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(6) NUMERICAL ASSESSMENT OF THE COMPUTER CODES FOR ANALYZING BOUNDARY LAYER TRANSITION ON A HEATED AXISYMMETRIC BODY

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(10) By: Kent T. S./Tzou

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An assessment of the available analytical tools for analyzing boundary-layer transition on a heated axisymmetric body has been performed. The approach is focused on the evaluation of the validity and efficiency of the three major components in the TAPS Code, Axisymmetric Potential Flow, Boundary Layer Flow, and Linear Stability Analysis. In addition, further developments of the TAPS Code, the incorporation of roughness model is also presented. Several conclusions on the code comparison have been made. For Axisymmetric Potential Flow, the

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20. ABSTRACT (Continued)

numerical approach of Landweber for solving the Fredholm integral equation of the first kind is relatively simple and less time consuming when compared with the approach of Hess and Smith for solving the Fredholm integral equation of the second kind. The results from both codes are all sufficiently accurate for well-rounded bodies. For bodies with sudden changes in slope and curvature, or with local bumps, the approach of Hess and Smith can provide a better approximation than Landweber's ~~approach~~ for potential flow calculations. 13

For Boundary-Layer Flow calculation, a new code developed by Cebeci appears to have merit for improving computational efficiency. The key factor is attributed to the fact that it solves the momentum and energy equations simultaneously. Results from both codes are in good agreement.

For the purpose of Spatial Amplification Analysis, the stability component of the TAPS Code is less expensive than the code developed by Lowell and Reshotko. Since the validity of partial non-parallel flow effect in the TAPS Code is questionable and the computer code to account for the effect of full non-parallel flow with heat transfer is not available at ~~the~~ present ~~time~~, the parallel flow option in the TAPS Code is recommended. But, bear in mind that some degree of conservatism should be taken when applying parallel flow stability results to growing boundary layers.

The applicability of the roughness model in the TAPS Code still requires more reliable experimental data. However, this model can be used for a first-order estimate of the effect of distributed roughness on the boundary-layer characteristics for designing a heated underwater vehicle.



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## 1. INTRODUCTION

In recent years, tremendous research efforts have been focused on the development of a low drag underwater vehicle. Among many techniques for achieving the low drag objective, the use of heating on the body surface for delaying the onset of boundary-layer transition, has received particular attention. The design of such low-drag vehicles relies critically on the ability of predicting various phenomena which might affect the transition location. For many years, the "$e^9$" method proposed by Smith (1956) has been considered as adequate for estimating the transition location on a smooth body over a range of parameters for which data exist. At the present time, the most commonly used engineering design tool is the so-called TAPS Code (Transition Analysis Program System) developed and updated by the McDonnell Douglas Corporation (Gentry, *et al.* 1977, Lee, *et al.* 1978). This rather large computer program (approximately 17,250 source cards) contains the following six major components:

- Geometry
- Two-Dimensional Potential Flow
- Axisymmetric Inverse Potential Flow
- Axisymmetric Potential Flow
- Boundary Layer Calculation
- Stability Analysis

For engineering applications, the procedure for designing a heated underwater vehicle can be described by a flow diagram as shown in Figure 1. The last three components in the TAPS Code are the essential tools for the analytical computations. It is noted here that "$e^n$" instead of "$e^9$" is used for the design criterion because the TAPS Code does not include the effects of several potentially important factors such as free stream turbulence, surface roughness, surface waviness, buoyancy forces, suspended particulates, etc., which are present in a realistic environment. In order to provide some degree of conservatism, n = 5 or 6 is commonly used as a design criterion.

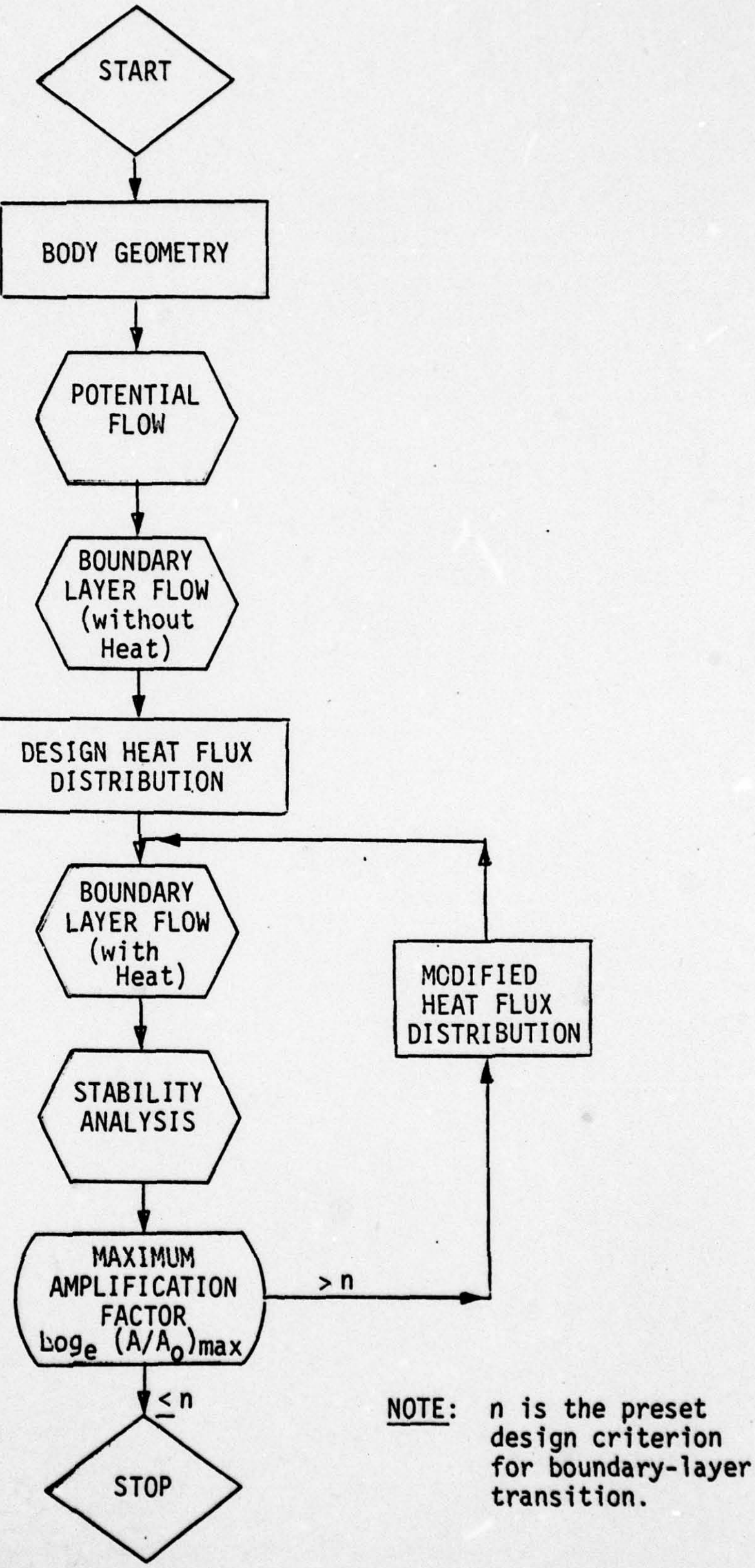



Figure 1. Flow Diagram for Designing a Heated Laminar Flow Body

In this study, an attempt was made to perform a numerical assessment of three major components in the TAPS Code, namely: Axisymmetric Potential Flow, Boundary-Layer Flow and Stability Analysis. In addition, a further development of the TAPS Code - the incorporation of a roughness model - is also included. A brief description of the distinct features of the components of the TAPS Code and other codes which will be used for comparison is presented below.

Potential Flow Calculation

(a) TAPS (Hess & Smith, McDonnell Douglas Corp.)

- Uses source distribution on body surface.
- Solves Fredholm integral equation of the second kind.

(b) IOWA (Landweber, University of Iowa)

- Uses doublet distribution on body axis.
- Solves Fredholm integral equation of the first kind.

Boundary Layer Flow Calculation

(a) TAPS (Cebeci & Smith, McDonnell Douglas Corp.)

- Uses Probstein-Elliott and Levy-Lees transformations.
- Solves momentum and energy equations by an iterative method.

(b) MDCBL (Cebeci *et al.*, 1978, McDonnell Douglas Corp.)

- Uses Mangler and Falkner-Skan transformations.
- Solves momentum and energy equations simultaneously.

Linear Stability Analysis

(a) TAPS (Wazzan & Smith, McDonnell Douglas Corp.)

- Uses Wazzan's modification of the Orr-Sommerfeld (O-S) equation for water to include the effect of heat transfer.

- Solves this modified O-S equation by using a Runge-Kutta scheme together with the Gram-Schmidt orthogonalization procedure.

(b) CWRU (Lowell & Reshotko, Case Western Reserve University)

- Uses coupled vorticity and energy disturbance equations to include the effects of heat transfer, viscosity and temperature fluctuations.

- Solves these equations by the same integration method as TAPS.

The main objective of this report is to provide a systematic and independent component-by-component comparison and validation of the TAPS code in order to lend further credence to the utility of that code for future design application. This report, hopefully, will serve as a valuable supplementary document for future TAPS users and low-drag body designers.

Discussions on the potential flow calculation, boundary-layer flow calculation and linear stability analysis are presented in Sections 2 through 4 respectively. Further development of the TAPS code is discussed in Section 5. A summary and several conclusions based on the current study are in Section 6.

## 2. POTENTIAL FLOW CALCULATION

In the evaluation of the hydrodynamic performance of an underwater vehicle, potential flow calculations for obtaining velocity or pressure distribution on the body surface are essential for the subsequent boundary layer computation and stability analysis. The most commonly used computer program for potential flow calculation is the so-called Douglas Neumann Program developed by Hess and Smith (1967). The method utilizes a source distribution over the body surface and computes this distribution as the solution of a Fredholm integral equation of the second kind. This method has been constructed for two-dimensional, axisymmetric and fully three-dimensional shapes.

Another analytical approach to the potential flow problem has been formulated and developed by Landweber (1951) for two-dimensional and axisymmetric bodies. This method has not been widely used in engineering applications because it has no commercial user's manual available. The approach consists of formulating the potential flow problem as a Fredholm integral equation of the first kind by using an axial distribution of doublets.

The axisymmetric potential-flow component of the TAPS Code has been derived from the Douglas Neumann Program by Gentry (1976). A brief description of the different approaches of Hess-Smith and of Landweber will be discussed in the following section.

## 2.1 Analytical Approach of Hess and Smith (TAPS)

The total potential function $\Phi$ for a uniform incompressible inviscid flow passing through an arbitrary body surface as shown in Figure 2 can be expressed as the sum of two components (Hess & Smith, 1967)

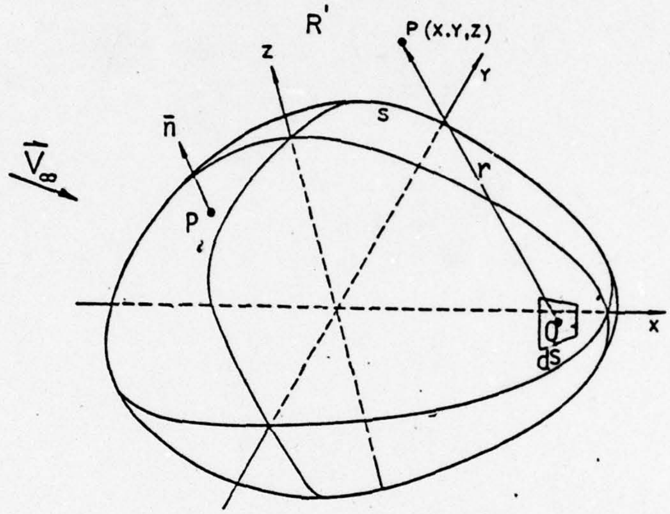



Figure 2. Flow About an Arbitrary Body Surface (from Hess & Smith, 1962)

$$\Phi = \phi_\infty + \phi \qquad (1)$$

where $\phi_\infty$ is the potential function of the oncoming uniform stream and $\phi$ is the disturbance potential function due to the body. The function $\Phi$ satisfies Laplace's equation in the region R' exterior to S, has a zero normal derivative on S, and approaches the proper uniform stream potential at infinity. In terms of velocity vectors, equation (1) leads to

$$\bar{V} = \bar{V}_\infty + \bar{v} \qquad (2)$$

where $\overline{V}_\infty$ is the incoming uniform flow velocity and $\bar{v}$ is the disturbance velocity field due to the body. It is assumed that $\overline{v}$ is an irrotational flow field and its disturbance potential function $\phi$ satisfies Laplace's equation in the region R'.

$$\overline{v} = - \nabla\phi \tag{3}$$

$$\nabla^2\phi = 0 \tag{4}$$

The boundary conditions on $\phi$ is

$$\left.\frac{\partial\phi}{\partial n}\right|_S = \overline{V}_\infty \cdot \left.\overline{n}\right|_S - F \tag{5}$$

where $\overline{n}$ is the unit normal vector to a surface and F is the prescribed normal velocity on boundary surface.

In the exterior problem, the boundary condition is

$$|\nabla\phi| \longrightarrow 0 \quad \text{at infinity} \tag{6}$$

As shown in Figure 2, consider the potential of a continuous source distribution m (Q) on the surface S, where the source point Q now denotes a general point on the surface S, then the disturbance potential of the distribution is

$$\phi = \iint_S \frac{m(Q)}{r(P,Q)}\, dS \tag{7}$$

In accordance with the procedure presented by Kellogg (1929), the result is the following integral equation for the source-density distribution m(P).

$$2\pi m(P) - \iint_S \frac{\partial}{\partial n} \left(\frac{1}{r(P,Q)}\right) m(Q)\, dS = - \bar{n}(P) \cdot \bar{V}_\infty + F \qquad (8)$$

This is a Fredholm integral equation of the second kind over the boundary surface S. Once this equation is solved for the source distribution, m(P), then the velocity potential may be evaluated from equation (7) and the disturbance flow velocity from equation (3). The corresponding pressure coefficient $C_p$ can be obtained by

$$C_p = 1 - \frac{|\bar{V}|^2}{|\bar{V}_\infty|^2} \qquad (9)$$

To implement this method, the body is approximated by a large number of small surface segments with constant source density. The integral equation (8) then is replaced by a set of linear algebraic equations for the values of the source density on the segments. Details of the numerical approach can be found in the papers by Hess & Smith (1962, 1964, 1967, 1972, 1974, 1975).

## 2.2 Analytical Approach of Landweber

The alternative method proposed by Landweber (1951, 1959) is suitable for computing the potential flow for simple bodies of revolution that are simultaneously translating at some angle to the axis of symmetry and rotating about a transverse axis. In this study, only the simplest case, an axial motion of a body will be presented.

As shown in Figure 3, the x-axis coincides with the axis of the body of revolution; the body extends along its axis from x = -1 to x = +1.

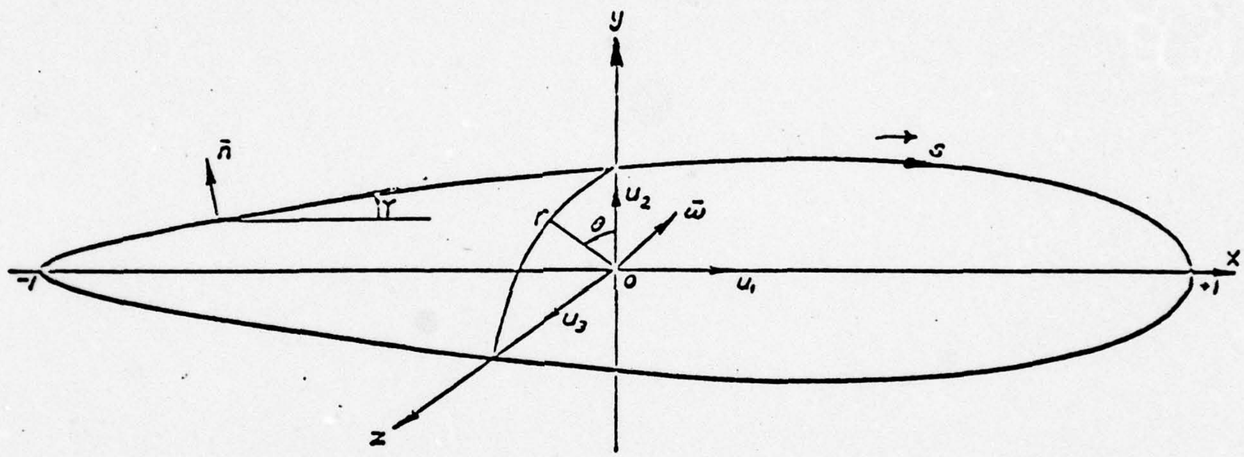



Figure 3. Flow About a Body of Revolution
(from Landweber, 1959)

Let $\phi$ and $\phi'$ be any two functions harmonic in the region exterior to the body and vanishing at infinity. Then Green's reciprocal theorem gives

$$\iint \phi \frac{\partial \phi'}{\partial n} dS = \iint \phi' \frac{\partial \phi}{\partial n} dS \qquad (10)$$

where the double integrals are taken over the surface of the body. For an axial motion, Landweber assumed that $\phi'$ is the doublet potential function $\left(\frac{x - \xi}{R^3}\right)$ and $\phi$ is the axisymmetric potential function.

More specifically,

$$\phi' = \frac{x - \xi}{R^3} \quad ; \quad R = \left[(x-\xi)^2 + r^2\right]^{1/2} \tag{11}$$

where $\xi$ is the location of doublet on the x-axis. The corresponding Stokes stream function for $\phi'$ is

$$\psi' = \frac{r^2}{R^3} \quad . \tag{12}$$

In terms of the cylindrical coordinates $(x, r, \Theta)$, the surface element can be expressed as

$$dS = 2\pi r ds \tag{13}$$

where s is the arc length along a meridian section measured clockwise from the point x = -1 to x = +1 with 2p as the perimeter of the section. Furthermore

$$\frac{\partial\phi}{\partial n} = - \sin \alpha = - \frac{\partial r}{\partial s} \tag{14}$$

then equation (10) becomes

$$\int_0^p r\left[\phi \frac{\partial}{\partial n} \left(\frac{x-\xi}{R^3}\right) + \left(\frac{x-\xi}{R^3}\right) \frac{\partial r}{\partial s}\right] ds = 0 \tag{15}$$

From the relationship between an axisymmetric potential and its Stokes stream function

$$r \frac{\partial}{\partial n} \left(\frac{x-\xi}{R^3}\right) = \frac{\partial}{\partial s} \left(\frac{r^2}{R^3}\right) \quad . \tag{16}$$

Integration by parts yields

$$\int_0^p \phi r \frac{\partial}{\partial n} \left(\frac{x-\xi}{R^3}\right) ds = \int_0^p \phi \frac{\partial}{\partial s} \left(\frac{r^2}{R^3}\right) ds = - \int_0^p \frac{r^2}{R^3} \frac{\partial \phi}{\partial s} ds \quad . \tag{17}$$

Now let $\Phi$ be the total potential function when the flow is made steady by superimposing a uniform stream of unit velocity in the negative x-direction. Then

$$\Phi = \phi - x \tag{18}$$

Hence

$$\frac{\partial \phi}{\partial s} = \frac{\partial \Phi}{\partial s} + \frac{\partial x}{\partial s} \tag{19}$$

and equation (15) becomes

$$\int_0^p \left[- \frac{r^2}{R^3} \left(\frac{\partial\Phi}{\partial s} + \frac{\partial x}{\partial s}\right) + r \left(\frac{x-\xi}{R^3}\right) \frac{\partial r}{\partial s}\right] ds = 0 \tag{20}$$

or

$$\int_0^p \frac{\partial\Phi}{\partial s} \frac{r^2}{R^3} ds = \int_0^p \left[- \frac{r^2}{R^3} dx + r \frac{(x-\xi)}{R^3} dr\right] \tag{21}$$

From the identity

$$- \frac{r^2}{R^3} dx + r \frac{(x-\xi)}{R^3} dr = - d \left(\frac{x-\xi}{R}\right) \tag{22}$$

equation (21) becomes

$$\int_0^p \frac{\partial\Phi}{\partial s} \frac{r^2}{R^3} ds = -2 \tag{23}$$

putting $ds = dx \sec\gamma$ and denoting the tangential velocity $\frac{\partial\Phi}{\partial s} = V_s$, equation (23) becomes

$$\int_{-1}^1 V_s \sec \gamma \frac{r^2}{R^3} dx = -2 \tag{24}$$

This is a Fredholm integral equation of the first kind for the unknown tangential velocity $V_s$ on the body surface. Once this equation is solved for $V_s$, the pressure distribution $C_p$ can be calculated accordingly.

Procedures for solving Fredholm integral equations of the first kind by an iterative method have been discussed in detail by Landweber in his publications (1951, 1959).

## 2.3 Comparison of the Calculated Pressure Coefficient Distribution

As shown in Figure 4, a vehicle with geometry designated as "Body A" was used as a test case for the comparison of two codes. It is noted that the resulting pressure coefficients from both codes are in remarkable agreement.

## 2.4 Comparison of the Computational Efficiencies

The required computational time and cost for processing the programs on a CDC 7600 Computer are listed as follows:

CDC 7600 Computer

(96 Stations on the Body Coordinates)

| | SRU | Cost |
|---|---|---|
| TAPS | 6.06 sec | $4.24 |
| IOWA | 1.58 sec | $1.11 |

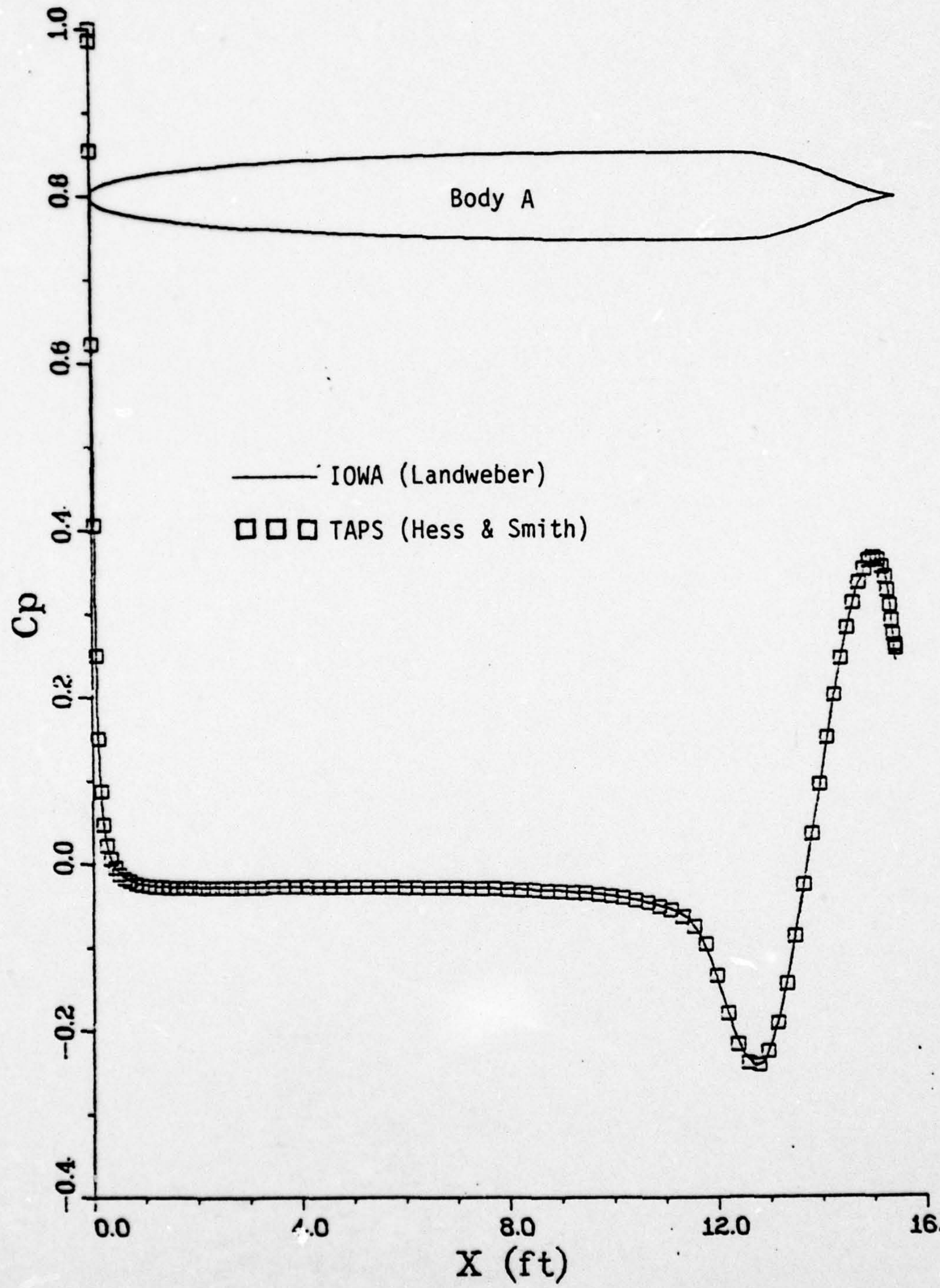



Figure 4. Comparison of the Calculated Pressure Coefficient Distributions from TAPS and IOWA.

where SRU is the System Resource Unit. For a normal priority run, it costs \$0.70 per SRU from the ITEL CDC 7600 System.

For this sample case, the computation using the IOWA Code costs substantially less than that using the TAPS Code. This is because the Fredholm integral equation of the first kind in equation (24) has a much simpler kernel than its second kind counterpart of equation (8). According to Landweber (1959), experience with a large number of bodies of revolution indicates that, for well-rounded bodies, sufficiently accurate solutions can be obtained without difficulty by means of integral equations of the first kind. For bodies with sudden changes in slope and curvature, or with local bumps, the method using Fredholm integral equations of the second kind continues to remain applicable and have been remarkably successful (Smith, 1958).

## 3. BOUNDARY LAYER FLOW CALCULATIONS

After having the velocity or pressure distribution on the body surface, the next step is the calculation of laminar boundary layer development. The governing boundary-layer equations for steady axisymmetric compressible laminar boundary layers are the continuity, momentum, and energy equations. These equations and their boundary conditions for the coordinate system shown in Figure 5 are:

Continuity

$$\frac{\partial}{\partial x}(r\rho u) + \frac{\partial}{\partial y}(r\rho v) = 0 \tag{25}$$

Momentum

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} + \frac{1}{r}\frac{\partial}{\partial y}\left[r\left(\mu\frac{\partial u}{\partial y}\right)\right] \tag{26}$$

Energy

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{1}{r}\frac{\partial}{\partial y}\left\{r\left[\frac{K}{c_p}\frac{\partial H}{\partial y} + \left(\mu - \frac{K}{c_p}\right) u \frac{\partial u}{\partial y}\right]\right\} \tag{27}$$

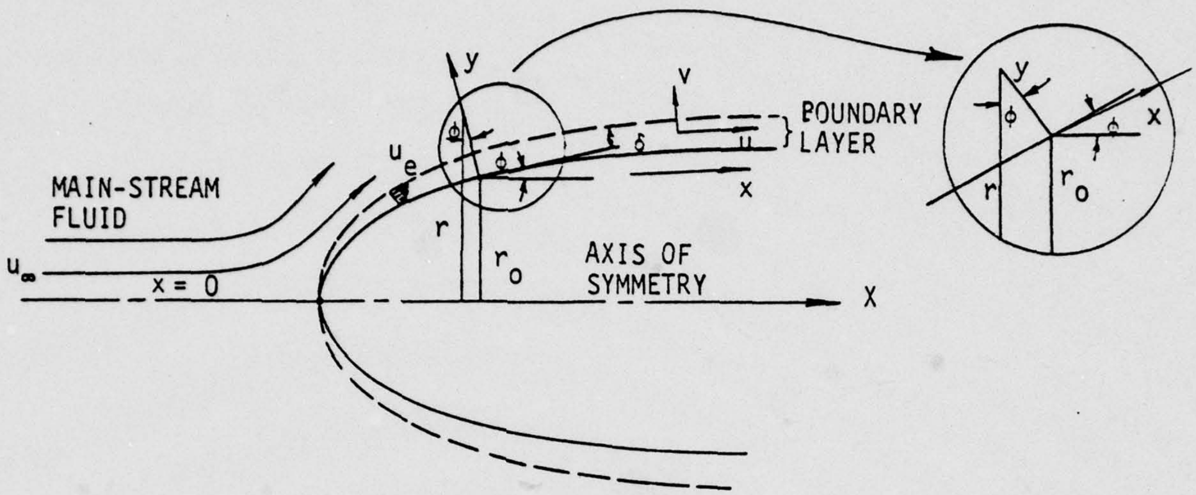



Figure 5. The Coordinate System for An Axisymmetric Boundary-Layer Flow (from Cebeci *et al.*, 1978)

$$
\begin{aligned}
&y = 0 \qquad u = 0\,,\quad v = v_w(x)\,,\quad H = H_w \text{ or } \left(\frac{\partial H}{\partial y}\right)_w = \text{given} \\
&y \longrightarrow \infty \qquad u \longrightarrow u_e(x)\,,\quad H \longrightarrow H_e
\end{aligned}
\tag{28}
$$

where $x$ = surface distance along the body surface

$y$ = distance normal to the surface

$X$ = axial distance

$u$ = streamwise ($x$) component of fluid velocity

$v$ = cross-stream ($y$) component of fluid velocity

$r$ = radial (or normal) distance from the axis

$\rho$ = density

$H$ = total enthalpy

$\mu$ = molecular viscosity

$K$ = thermal conductivity

$c_p$ = specific heat at constant pressure

e = conditions at the outer edge of the boundary layer

w = conditions at the wall

The new code developed by Cebeci *et al.* (1978) maintains the same governing equations and boundary conditions as the old one (Cebeci & Smith, 1974). A brief description of the two methods is presented below:

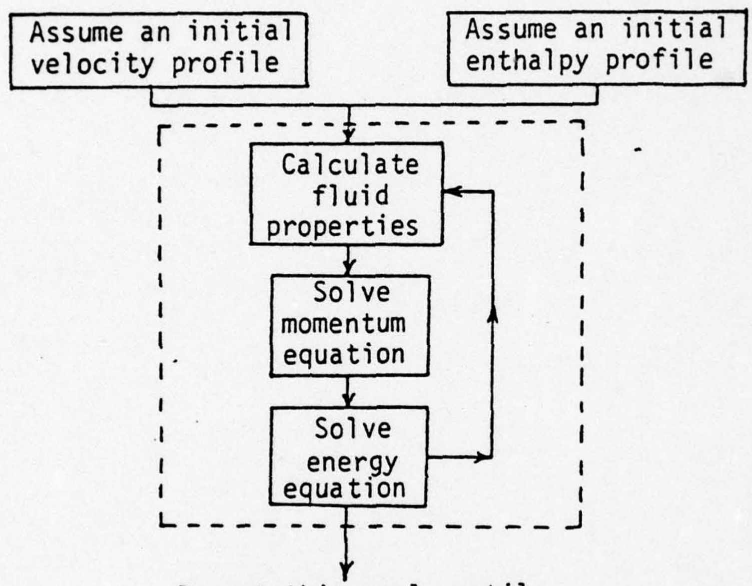
### 3.1 Method of Cebeci & Smith (TAPS)

The boundary layer component of the TAPS Code is based on the Cebeci-Smith finite difference program (Cebeci & Smith, 1974). To obtain the solution of boundary layer equations (25) to (28) the so-called "Keller Box Method" (Keller & Cebeci, 1970, 1971) is used. It is first necessary to assume initial velocity and enthalpy profiles. The calculations then proceed with an iterative procedure in order to satisfy a convergence criterion. In the TAPS Code, the velocity gradient as the wall $f''_w$ is used as the convergence control parameter and iteration is stopped when

$$\left| (f''_w)^{i+1} - (f''_w)^{i} \right| < \varepsilon$$

where i is the iteration number and $\varepsilon$ is a prescribed convergence criterion. A value of $\varepsilon = 0.005$ was recommended in the TAPS Code.

The general solution procedure used in the TAPS Code for boundary-layer flow calculation is shown in Figure 6.




Repeat this cycle until

$$|(f''_w)^{i+1} - (f''_w)^{i}| < \varepsilon$$

then proceed to the next station.

Figure 6. Flow Diagram for the Station Iteration Procedure for Boundary-Layer Flow Calculation in TAPS Code (from Gentry, 1976)

### 3.2 New Method of Cebeci *et al.*

Under the sponsorship of DARPA, a new boundary-layer code has been developed by Cebeci, *et al.* (1978). The distinct differences in numerical approach are as follows:

- A combination of Mangler and Falkner-Skan transformations is used in the new code, while in the TAPS Code, a combination of Probstein-Elliott and Levy-Lees transformations was used.

- In the station iteration procedure, instead of solving the momentum equation and then the energy equation as shown in Figure 6, the new code solves the two equations simultaneously.

### 3.3 Comparison of the Calculated Boundary-Layer Characteristics

A vehicle designated as "Body B" with its geometry and pressure coefficient distribution as shown in Figure 7, was used as a sample case. The flow conditions are $U_\infty$ = 10.30 m/sec (or 20 Knots), $T_\infty$ = 19.44$^\circ$C (67$^\circ$F) with surface overheat temperature distribution shown in Figure 8. Figures 9 through 11 present the calculated boundary-layer characteristics from both codes. In general, results show fairly good agreement in the calculations of the displacement thickness, shape factor and local skin friction coefficient. The two codes even predict the same location for laminar separation, S = 3.89 m, for the sample case.

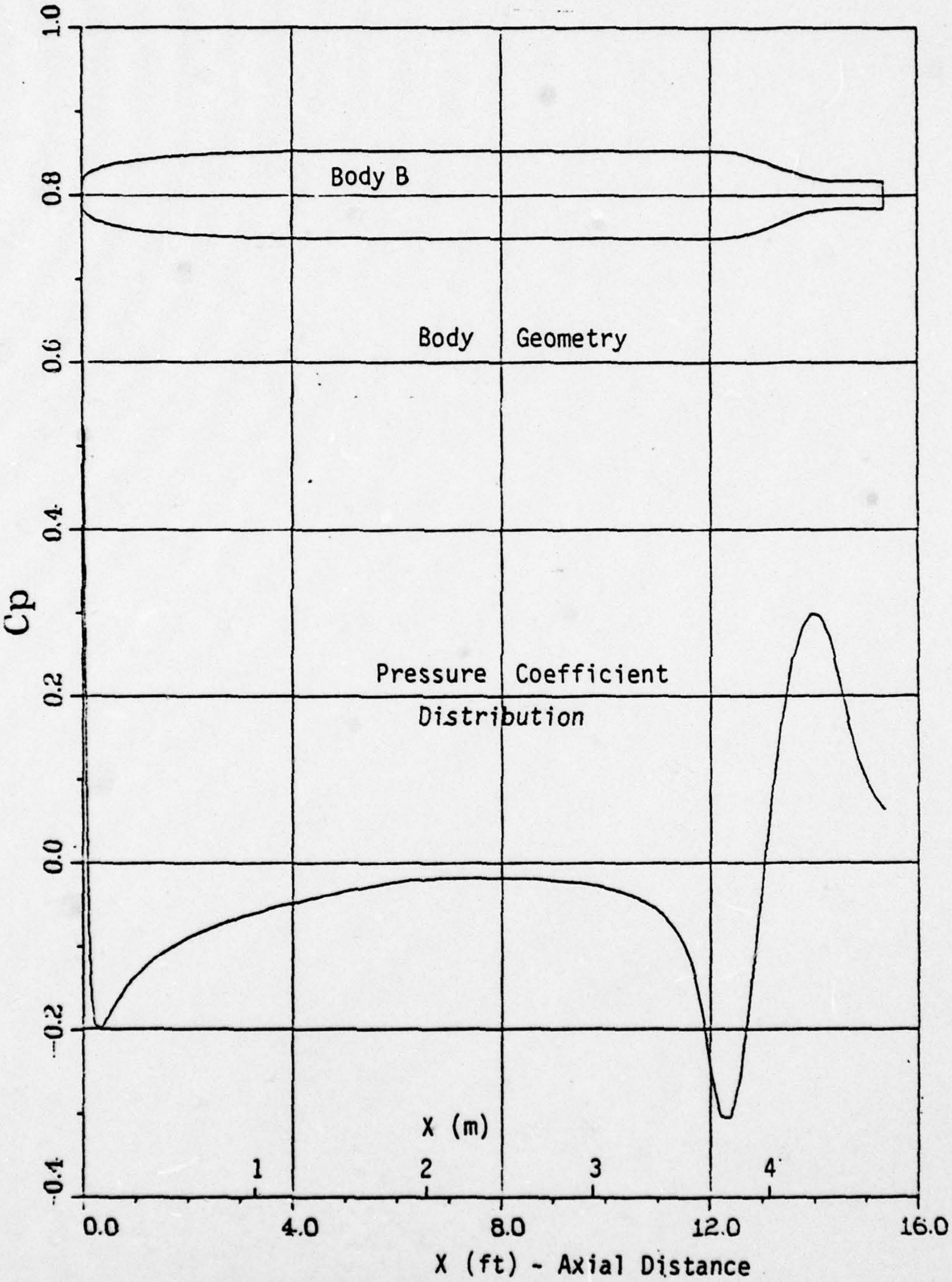



Figure 7. Body Geometry and Pressure Coefficient Distribution For Body "B".

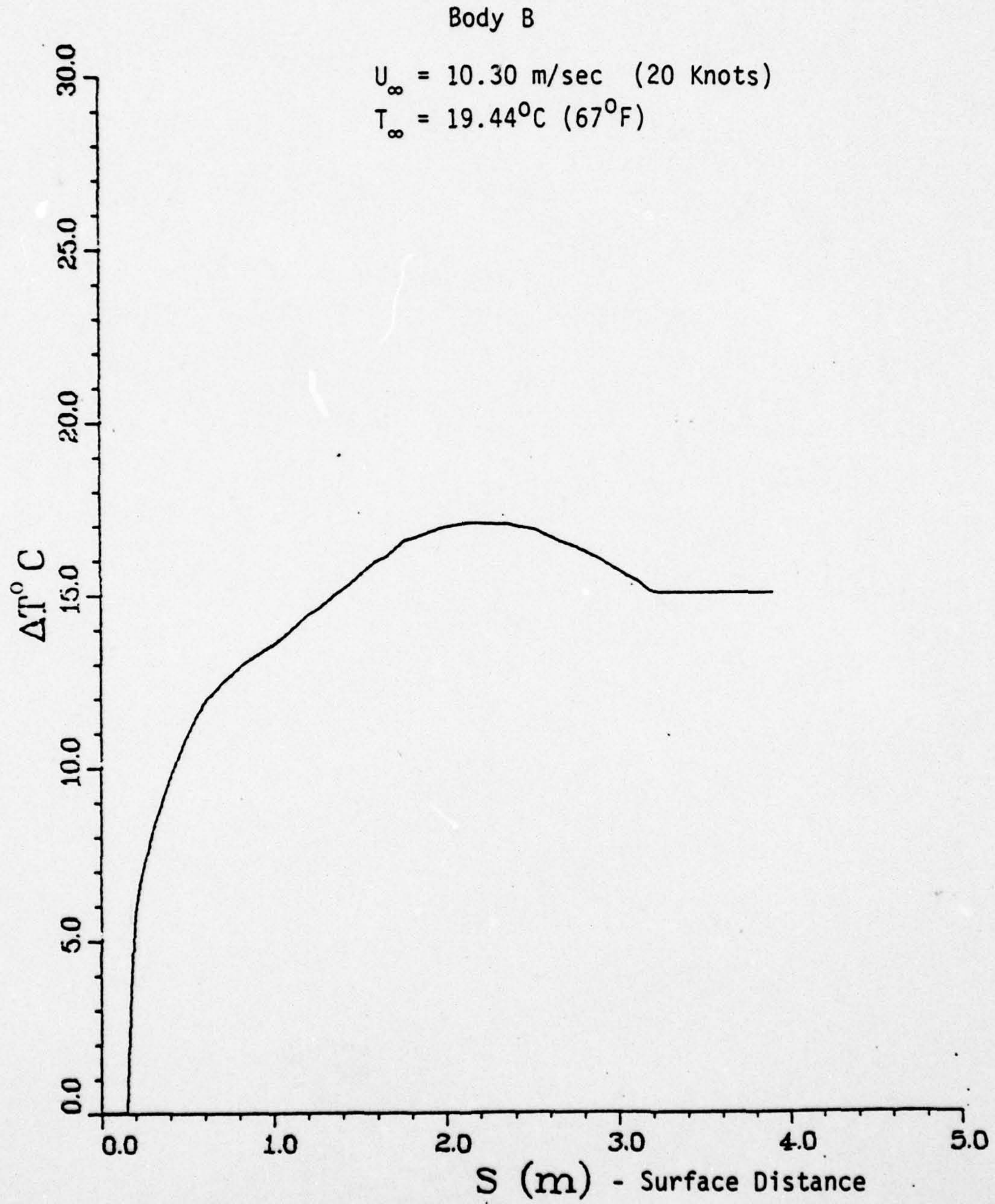



Figure 8. Distribution of Surface Overheat Temperature

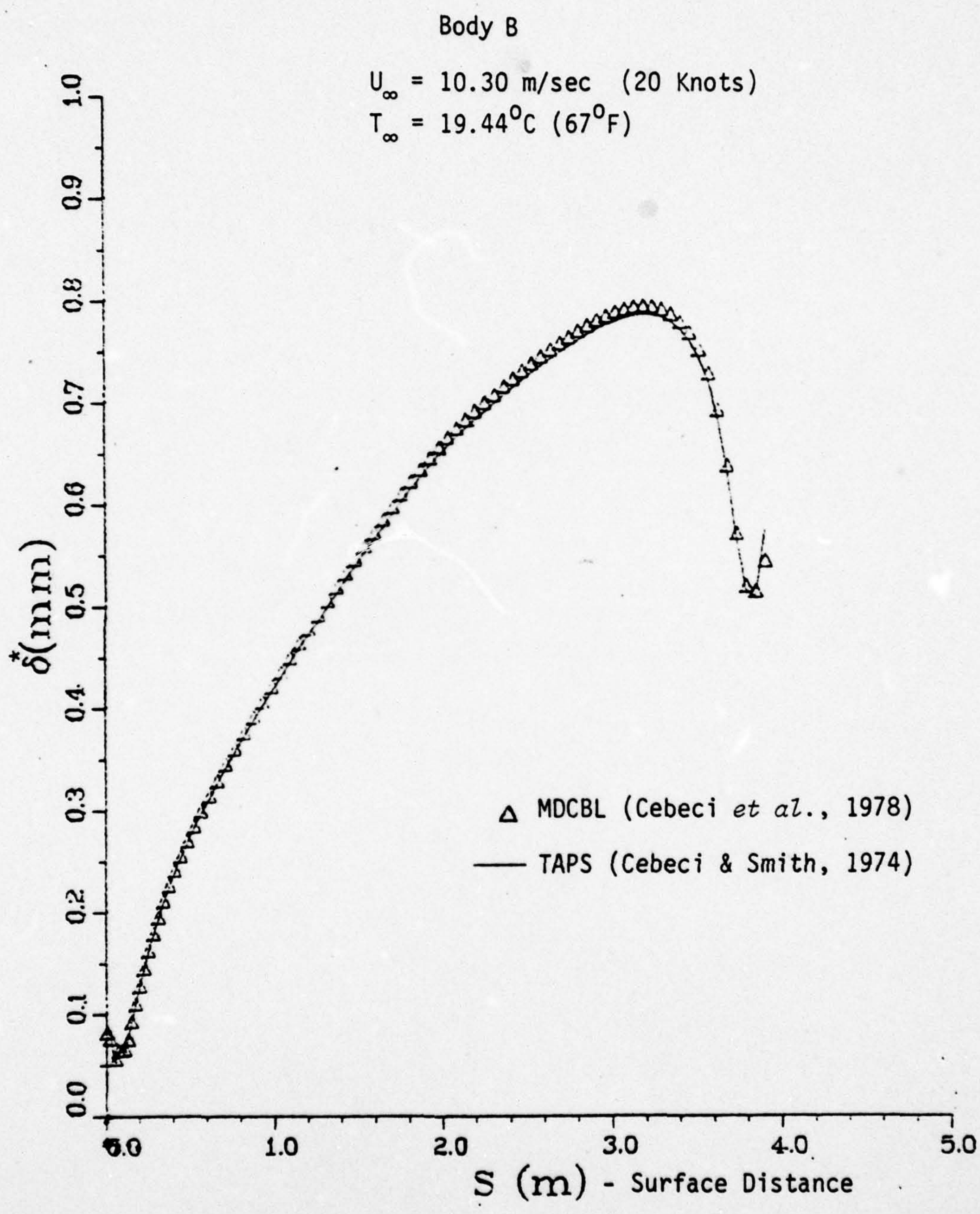



Figure 9. Comparison of Displacement Thickness, $\delta^*$.

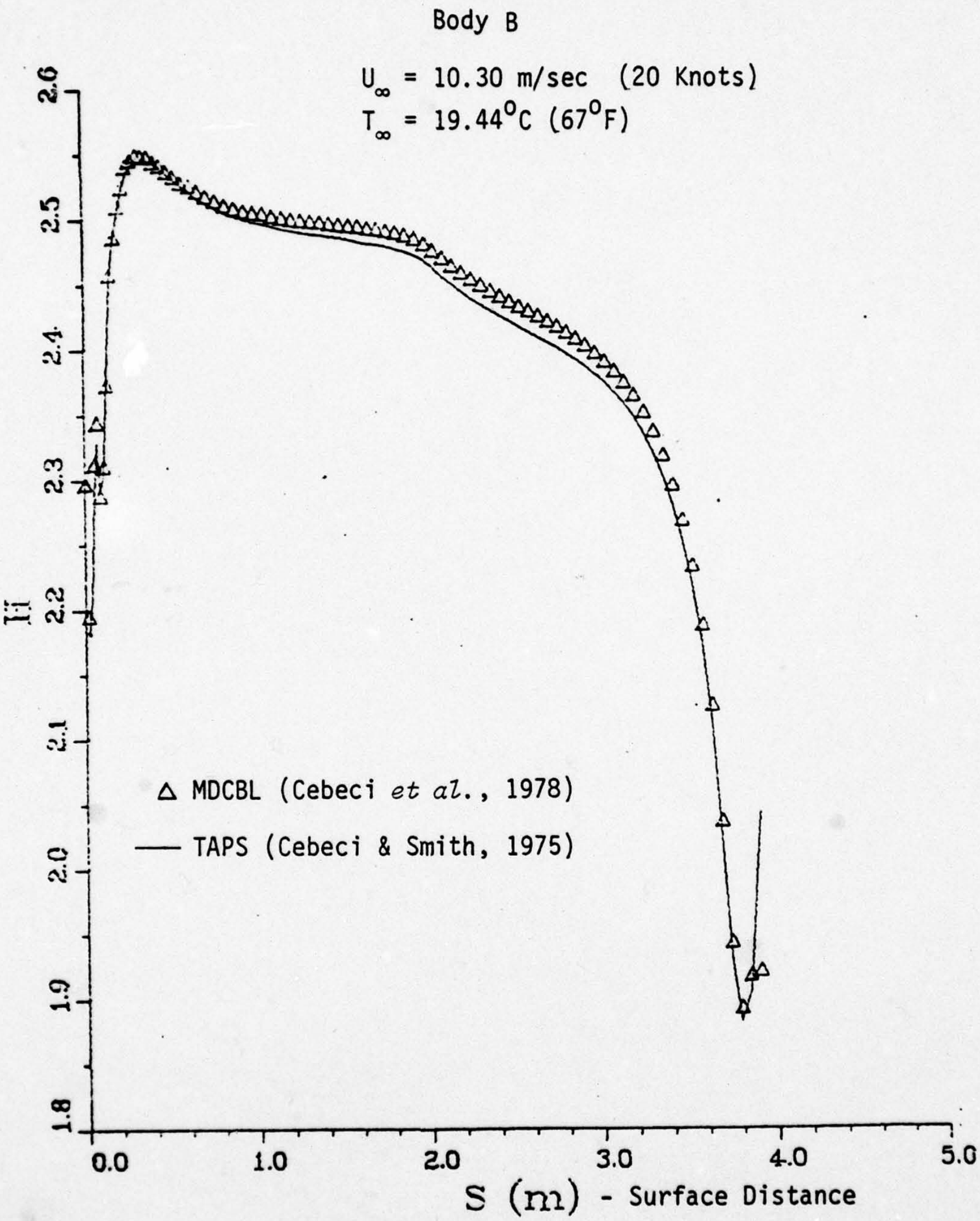



Figure 10. Comparison of Boundary-Layer Shape Factor, H.

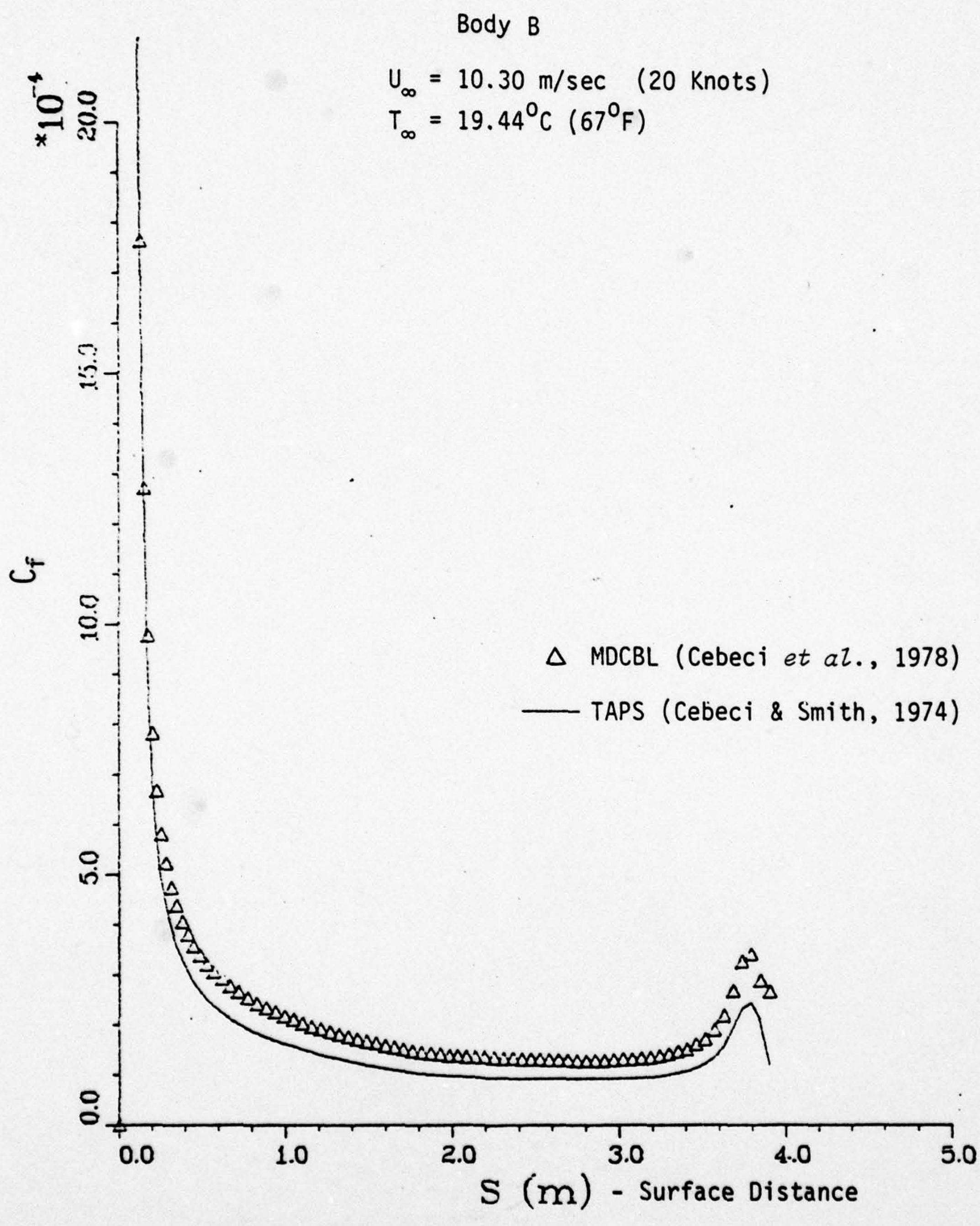



Figure 11. Comparison of Local Skin Friction Coefficient $C_f$.

## 3.4 Comparison of the Computational Efficiencies

The required computational time and cost for processing the programs on a CDC 7600 Computer are listed as follows:

CDC 7600 Computer

(82 Stations on the Body Coordinates)

| | SRU | Cost |
|---|---|---|
| TAPS | 12.00* | $8.40 |
| MDCBL | 5.13 | $3.59 |

* This is not including the option to save boundary layer profiles for the subsequent stability analysis. The computation time with that option is 25.00 SRU or $17.50.

The new code shows considerable reduction in computational time. This might be attributed to its improved numerical approach which solves the momentum and energy equations simultaneously.

## 4. LINEAR STABILITY ANALYSIS

In the analytical prediction of laminar flow transition, the so-called "$e^9$" method proposed by Smith (1956) has been considered as an adequate method for estimating the transition location on a smooth surface body. The method is based on the growth of spatial amplification factors from the linear stability analysis. A brief description of the approach is as follows:

The fundamental differential equation of a small traveling-wave disturbance in an incompressible constant property two-dimensional parallel laminar flow can be written as

$$(\overline{u}-\overline{c})\,(\phi''-\alpha^2\phi) - \overline{u}''\phi = -\frac{i}{\alpha R_{\delta*}}\,(\phi''''-2\alpha^2\phi''+\alpha^4\phi) \tag{29}$$

This is the classical Orr-Sommerfeld equation where the wave amplitude is taken to grow with time. The definitions of various symbols are listed below:

$\overline{u}$ = dimensionless longitudinal velocity component of the mean flow,

$\overline{c}$ = dimensionless complex phase velocity of Tollmien-Schlichting disturbance,

$\phi$ = disturbance amplitude function,

$\alpha$ = disturbance wave number,

$i$ = $\sqrt{-1}$

$R_{\delta*}$ = Reynolds number based on displacement thickness,

$'$ = differentiation with respect to coordinate normal to surface.

As described in detail by Smith (1956), the solution to the O-S equation (29) yields an eigenvalue relationship $G(R_{\delta *},\alpha,\bar{c}) = 0$. In a spatial amplification analysis, the disturbance differential equation looks the same except that the eigenvalue problem considers the wavenumber to be complex $\alpha = \alpha_r + i\alpha_i$ and the frequency $\omega$ (where $\omega = \alpha\bar{c}$) to be real. At each station along the body surface with a specified boundary-layer profile, an eigenvalue solution which describes a surface in the four-dimensional parameter space $(R_{\delta *},\alpha_r,\alpha_i,\omega)$ can be obtained. For a fixed Reynolds number $(R_{\delta *})$ and a disturbance frequency $(\omega)$, there exists a set of solutions consisting of wavenumber $(\alpha_r)$ and amplification rate $(\alpha_i)$. The values of $\alpha_i$ denote the growth character of the wave as follows:

$\alpha_i > 0$ , damped disturbance - stable

$\alpha_i = 0$ , neutral disturbance - neutrally stable

$\alpha_i < 0$ , amplified disturbance - unstable

The amplification rates $(\alpha_i)$ that are obtained from solutions of the O-S equation are then integrated for a fixed frequency $\omega$ with respect to the streamwise direction s from the point of neutral stability $(\alpha_i=0)$, yielding the natural logarithm of the amplification factor

$$\mathrm{Log}_e\,(A/A_o) = -\int \alpha_i ds \qquad (30)$$

where $A/A_o$ denotes the ratio of the amplitude of the final disturbance to the amplitude of the original disturbance at the lower neutral point. According to the Smith "$e^9$" criterion, boundary layer transition could occur at the streamwise location where this amplitude ratio reaches a value of "$e^9$" (or 8103).

### 4.1 Analytical Approach of Wazzan & Smith (TAPS)

The stability analysis component of the TAPS program is based on the approach of Wazzan, Okamura and Smith (1968, 1970). A brief discription of the approach is presented below:

#### Parallel Flow with Heat Transfer

In order to extend the stability theory of small disturbances to include the effects of heat transfer, equation (29) must be modified to account for variable fluid properties. By neglecting temperature fluctuations, assuming viscosity to be a function of temperature only, and taking all other fluid properties to be constant, Wazzan *et al.* (1968) obtain the following linearized small disturbance equation for parallel flow with heat transfer in water or liquids.

$$(\bar{u}-\bar{c})(\phi''-\alpha^2\phi) - \bar{u}''\phi = -\frac{i}{\alpha R_{\delta^*}}\Big[\bar{\mu}(\phi''''-2\alpha^2\phi''+\alpha^4\phi) + 2\bar{\mu}'(\phi'''-\alpha^2\phi') + \bar{\mu}''(\phi''+\alpha^2\phi)\Big] \tag{31}$$

where $\bar{\mu}$ is the dimensionless molecular viscosity in the basic flow. Equation (31) is the Orr-Sommerfeld equation augmented by two terms dependent on the non-uniformity of the mean viscosity with the boundary layer. It retains the fourth-order differential character of the O-S equation.

## Non-Parallel Flow Effects

One of the most common assumptions in the linearized stability analysis of laminar boundary layers is the parallel flow assumption. In reality, flow around a body of revolution has velocity components in the free-stream direction ($\bar{u}$) as well as in the normal direction ($\bar{v}$). Wazzan *et al.* derived a modified disturbance equation to account for the normal velocity distribution $\bar{v}$ and axial variation of the mean flow properties. This equation is given by

$$
\begin{aligned}
&(\bar{u}-\bar{c})(\phi''-\alpha^2\phi) - \bar{u}''\phi - \left(\frac{\delta^2}{\bar{u}_e}\frac{\partial^2\bar{u}}{\partial x^2}\right)\phi - \frac{i}{\alpha}\left(\frac{\delta^2}{\bar{u}_e}\frac{\partial^2\bar{u}}{\partial x\partial y}\right)\phi' \\
&- \frac{i}{\alpha}\bar{v}\,(\phi'''-\alpha^2\phi') + \frac{i}{\alpha}\left(\frac{\delta^2}{\bar{u}_e}\frac{\partial^2\bar{v}}{\partial x^2}\right)\phi' \\
&= -\frac{i}{\alpha R_{\delta *}}\Big[\bar{\mu}(\phi''''-2\alpha^2\phi''+\alpha^4\phi) + 2\bar{\mu}'\,(\phi'''-\alpha^2\phi') \\
&+ \bar{\mu}''\,(\phi''+\alpha^2\phi)\Big]
\end{aligned}
\tag{32}
$$

This equation is also a fourth order differential equation.

It must be noted here that the non-parallel flow analysis used by Wazzan in the TAPS Code is only a partial non-parallel flow analysis since effects of stream-wise variations of disturbance amplitude and wave number and some higher-order terms are neglected.

## 4.2 Analytical Approach of Lowell & Reshotko

Further development in the linear stability analysis has been carried out by Lowell & Reshotko (1974) to introduce viscosity fluctuation ($\hat{\mu}$) and the temperature fluctuation ($\hat{t}$) terms into the system of disturbance equations. The Lowell-Reshotko equations are:

$$(\bar{u}-\bar{c})\,(\phi''-\alpha^2\phi) - \bar{u}''\phi = -\frac{i}{\alpha R_{\delta*}}\Big[\bar{\mu}\,(\phi''''-2\alpha^2\phi''+\alpha^4\phi) + 2\bar{\mu}'\,(\phi'''-\alpha^2\phi') + \bar{\mu}''\,(\phi''+\alpha^2\phi)\Big] - \frac{1}{\alpha R_{\delta *}}\Big[(\hat{\mu}\bar{u}')'' + \alpha^2\hat{\mu}\bar{u}'\Big] \qquad (33)$$

coupled with a disturbance energy equation

$$i\hat{t}\,(\bar{u}-\bar{c}) + H'\phi = \frac{1}{\alpha R_{\delta*} P_r}(\hat{t}''-\alpha^2\hat{t}) \qquad (34)$$

where H' is the derivative of the nondimensional mean temperature and $P_r$ is the Prandt number. The coupled system is sixth-order with nominal homogenous boundary conditions. If one eliminates the fluctuation terms ($\hat{\mu}$) and ($\hat{t}$) in equation (33) it becomes identical to equation (29), the parallel-flow equation of Wazzan & Smith, and there is no longer any coupling with equation (34).

## 4.3 Comparison of the Calculated Spatial Amplification Factors

From the analytical approaches of Sections (4.1) and (4.2) a summary of available options in linear stability analysis from computer codes of Wazzan-Smith (TAPS) and Lowell-Reshotko (CWRU, Case Western Reserve University) is presented in Table 1.

Table 1. A Summary of Options in Linear Stability Analysis

| | Wazzan-Smith | | Lowell-Reshotko | | |
|---|---|---|---|---|---|
| | TAPS(A) | TAPS(B) | CWRU(A) | CWRU(B) | CWRU(C) |
| 1. Parallel Flow, 4th order system | X | | X | X | X |
| 2. Partial Non-Parallel Flow ($\overline{v}$ terms only) | | X | | | |
| 3. Viscosity & Temperature Fluctuations, 6th order system | | | | X | X |
| 4. Fluid Properties of K & S | X | X | X | X | |
| 5. Fluid Properties of T.G.K.P. | | | | | X |

<u>Note</u>: K & S - Kaups & Smith (1967)

T.G.K.P. - Touloukian *et al.* (1970), Giddseth *et al.* (1972), Korosi *et al.* (1968) and Powell (1958)

In order to perform a numerical assessment of the effects of various options in the linear stability analysis, the Body B with the same flow conditions used in Section 3 has been chosen for this study. Detailed boundary-layer profiles for input to the stability analysis were calculated by the TAPS Code. The resulting spatial amplification factors are presented in the following subsections:

### 4.3.1 Effects of Partial Non-Parallel Stability Calculation

In the TAPS Code, in addition to the parallel flow assumption, it has the option to include some of the non-parallel flow terms as shown in Equation (32). For the sample case, the resulting amplification factors from both equations (31) and (32) are presented in Figures 12 and 13, respectively. As shown in Table 2 and Figure 14, the deviation ranges from 1% to 14% with a mean value of 6% when non-parallel terms are partially included in the O-S equation.

Table 2. Effects of Non-Parallel Terms on the Linear Stability Analysis

| Non-Dimensional Frequency $\omega \times 10^5$ | Maximum Amplification Factors $\log_e (A/A_0)$ | | |
|---|---|---|---|
| | TAPS (A) | TAPS (B) | % Deviation From Mean Value |
| 4.00 | 2.45 | 2.69 | 5 |
| 3.50 | 3.05 | 3.36 | 5 |
| 3.00 | 3.73 | 4.06 | 4 |
| 2.50 | 3.87 | 4.24 | 5 |
| 2.00 | 3.84 | 5.07 | 14 |
| 1.75 | 4.46 | 5.14 | 7 |
| 1.50 | 4.66 | 5.14 | 5 |
| 1.25 | 4.71 | 4.61 | 1 |
| 1.00 | 4.56 | 4.72 | 2 |
| 0.90 | 4.76 | 5.26 | 5 |
| 0.80 | 4.75 | 5.37 | 6 |
| 0.70 | *4.78 | *5.38 | 6 |
| 0.60 | 4.55 | 5.23 | 7 |
| 0.50 | 3.73 | 4.23 | 6 |

*** Highest Amplification Factor** Ave. = 6%

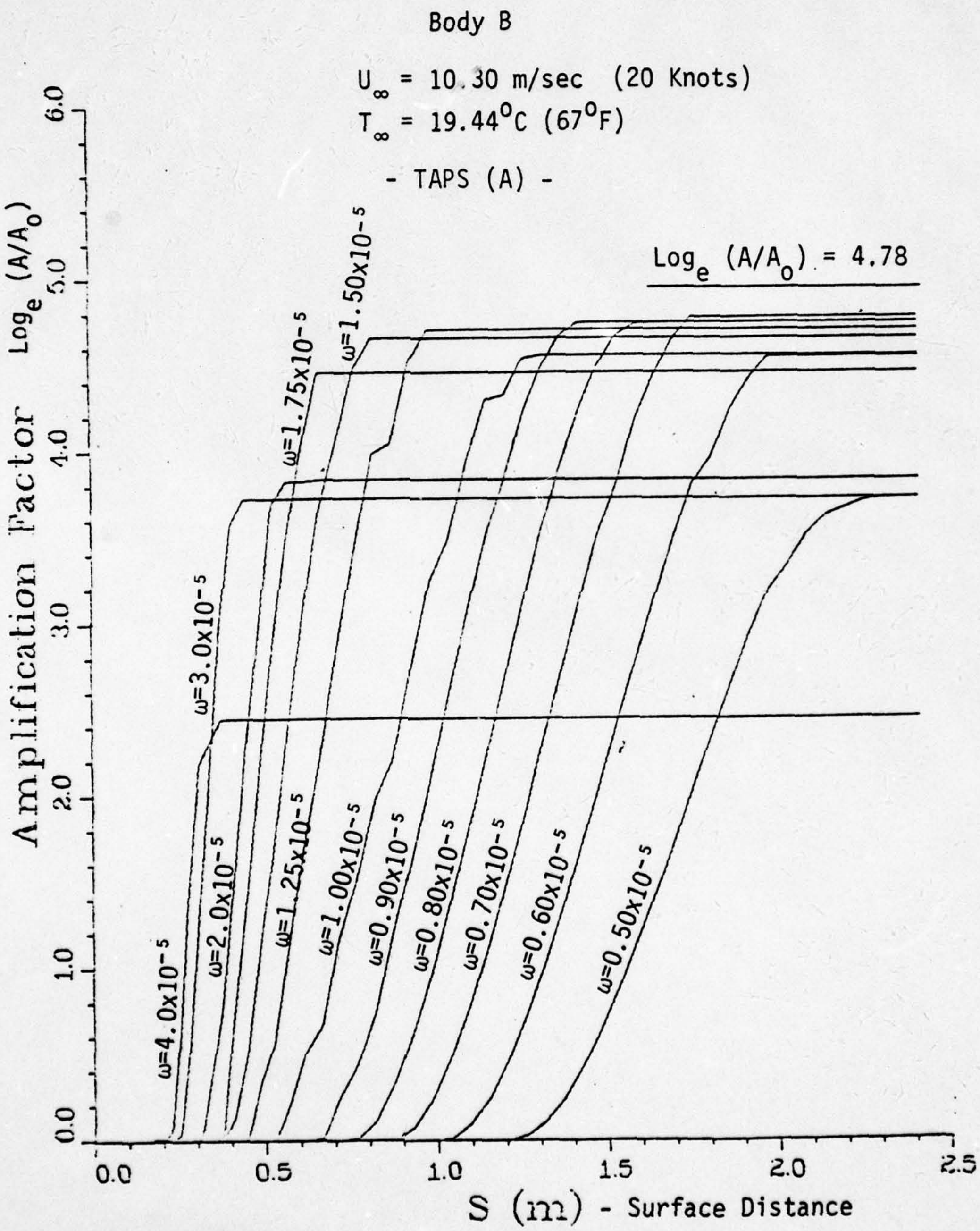



Figure 12. Spatial Amplification Factors In Boundary Layer, Method of Wazzan & Smith - Parallel Flow - TAPS (A)

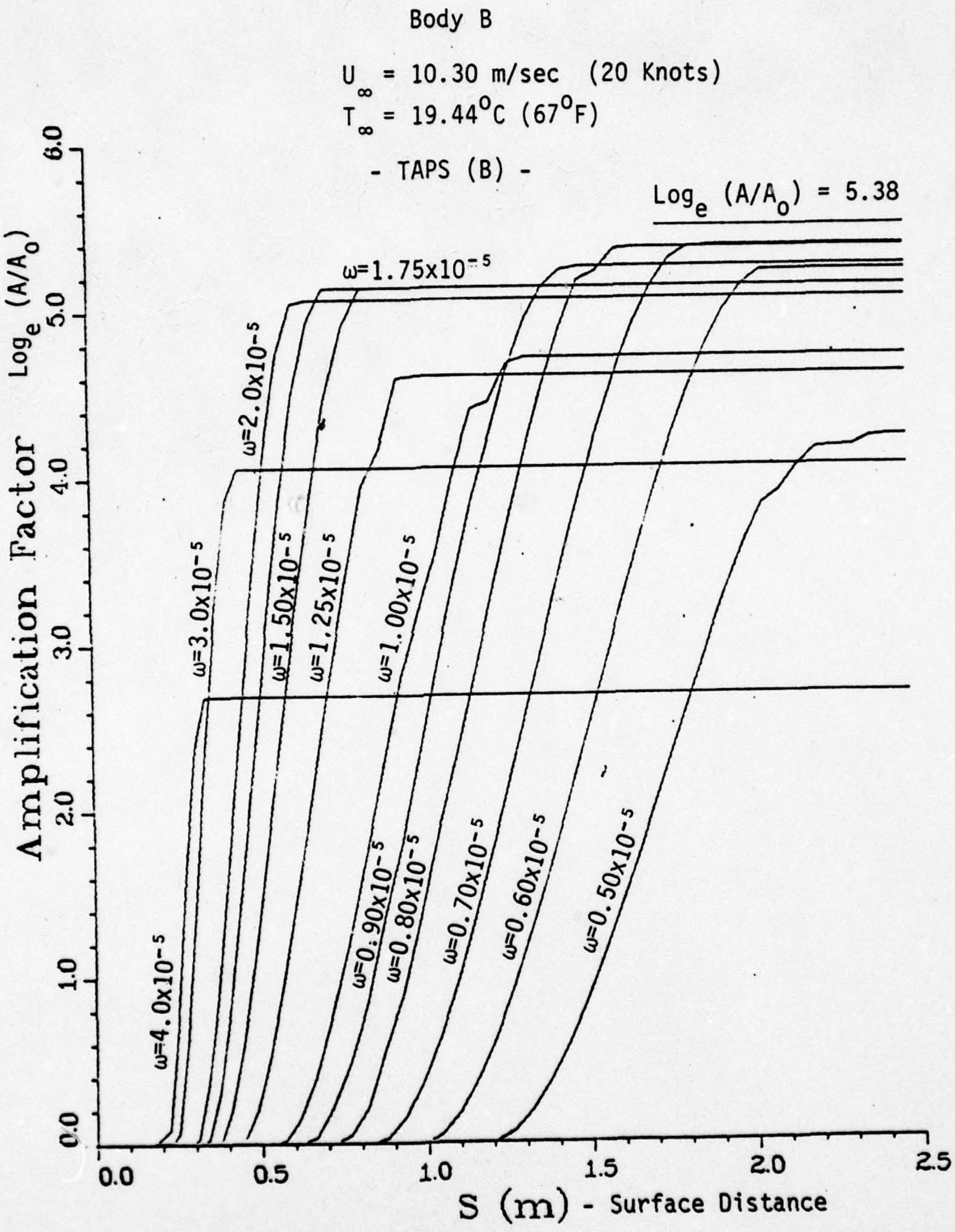



Figure 13. Spatial Amplification Factors in Boundary Layers, Method of Wazzan & Smith - Partial Non-Parallel Calculation - TAPS (B)

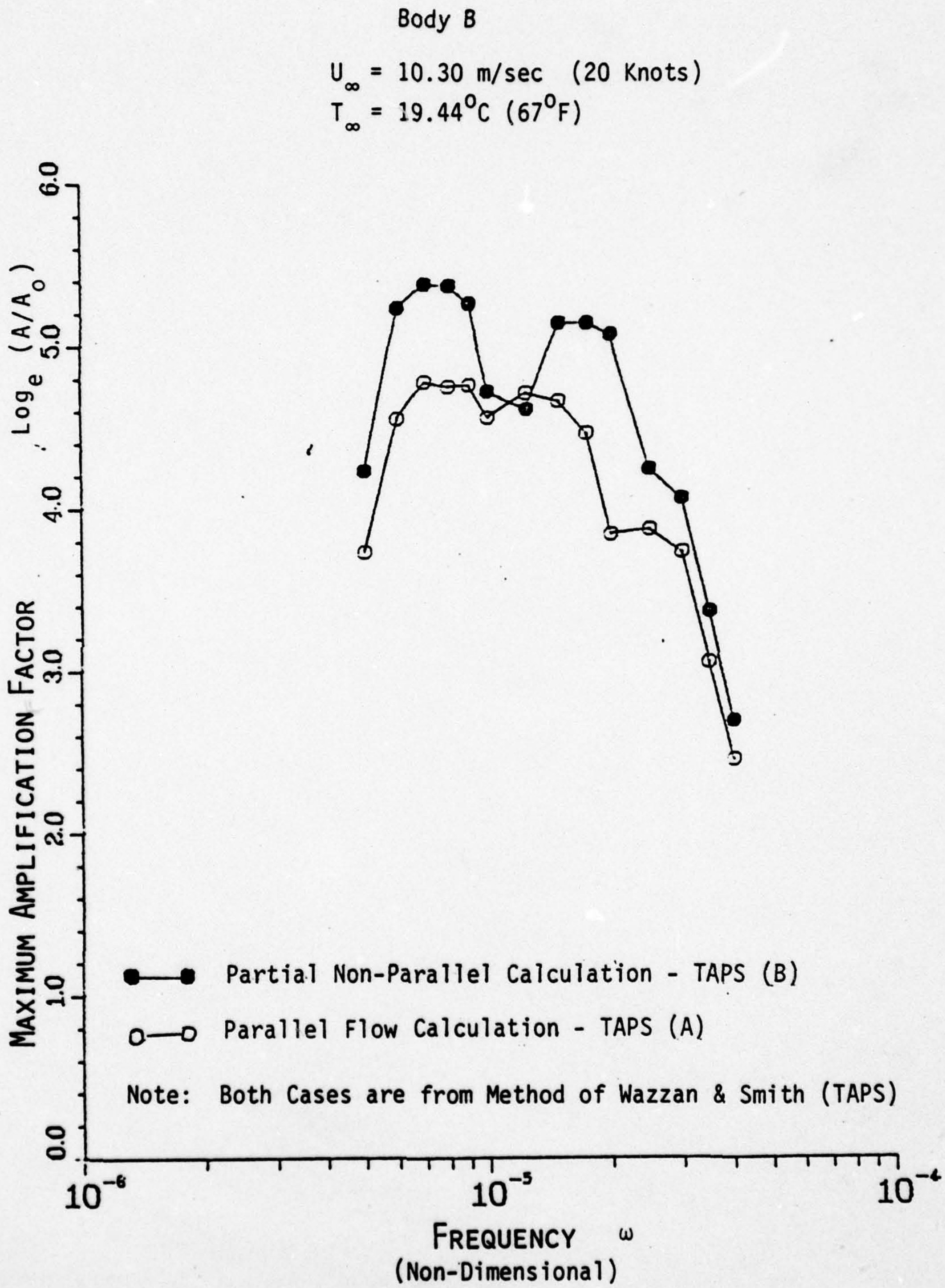



Figure 14. Effects of Non-Parallel Flow on the Linear Stability Analyses

A rather complete study of the effects of non-parallel stability has been carried out for constant property flows by Saric and Nayfeh (1975). The analysis takes into account the streamwise variations of the mean flow, the disturbance amplitude, and the wavenumber. For the Blasius flow, their calculated neutral curve based on the non-parallel theory agrees much better with the experimental data than the one calculated from the parallel flow theory. The growth rates from the non-parallel theory (Saric and Nayfeh, 1977) are also higher resulting in an amplification exponent n that for a number of examples is higher than the corresponding parallel flow value by about one. A very recent extension to uniformly heated plates in water by El-Hady and Nayfeh (1978) shows similar results, namely that non-parallel growth rates are higher than parallel flow growth rates and that the calculated non-parallel growth rates are in good agreement with the experimental results of Strazisar, Reshotko and Prahl (1977).

The parallel-flow results consistently underestimate the actual amplification factors if only slightly and so some degree of conservation should be taken when applying parallel flow results to growing boundary layers, at least until a proper non-parallel option is incorporated into the TAPS program.

### 4.3.2 Effects of Numerical Method of Solution

In the CWRU Code, if the viscosity and temperature fluctuation terms are eliminated, equation (33) is identical to equation (31), the equation used in the TAPS Code. Therefore, the main difference, if any, must be a result of the difference in the numerical method of solution. Figure 15 shows the resulting spatial amplification factors for the sample case from the CWRU Code. A comparison of the results from the two codes is presented in Table 3 and Figure 16. A rather large deviation in some of the frequencies has been observed, but the average value is only 6%.

Table 3. Effects of Numerical Method of Solution

| Non-Dimensional Frequency | Maximum Amplification Factors $\log_e (A/A_0)$ | | |
|---|---|---|---|
| | TAPS (A) | CWRU (A) | % Deviation from Mean Value |
| 4.00 | 2.45 | 1.66 | 19 |
| 3.50 | 3.05 | 2.42 | 12 |
| 3.00 | 3.73 | 3.19 | 8 |
| 2.50 | 3.87 | 4.12 | 3 |
| 2.00 | 3.84 | 4.69 | 10 |
| 1.75 | 4.46 | 4.91 | 5 |
| 1.50 | 4.66 | 4.91 | 3 |
| 1.25 | 4.71 | 4.91 | 2 |
| 1.00 | 4.56 | 4.88 | 3 |
| 0.90 | 4.76 | 4.89 | 1 |
| 0.80 | 4.75 | 4.98 | 2 |
| 0.70 | *4.78 | *5.08 | 3 |
| 0.60 | 4.55 | 5.01 | 5 |
| 0.50 | 3.73 | 4.15 | 5 |

* Highest Amplification Factor    Ave. = 6%

Body B

$U_\infty$ = 10.30 m/sec (20 Knots)
$T_\infty$ = 19.44°C (67°F)

- CWRU (A) -

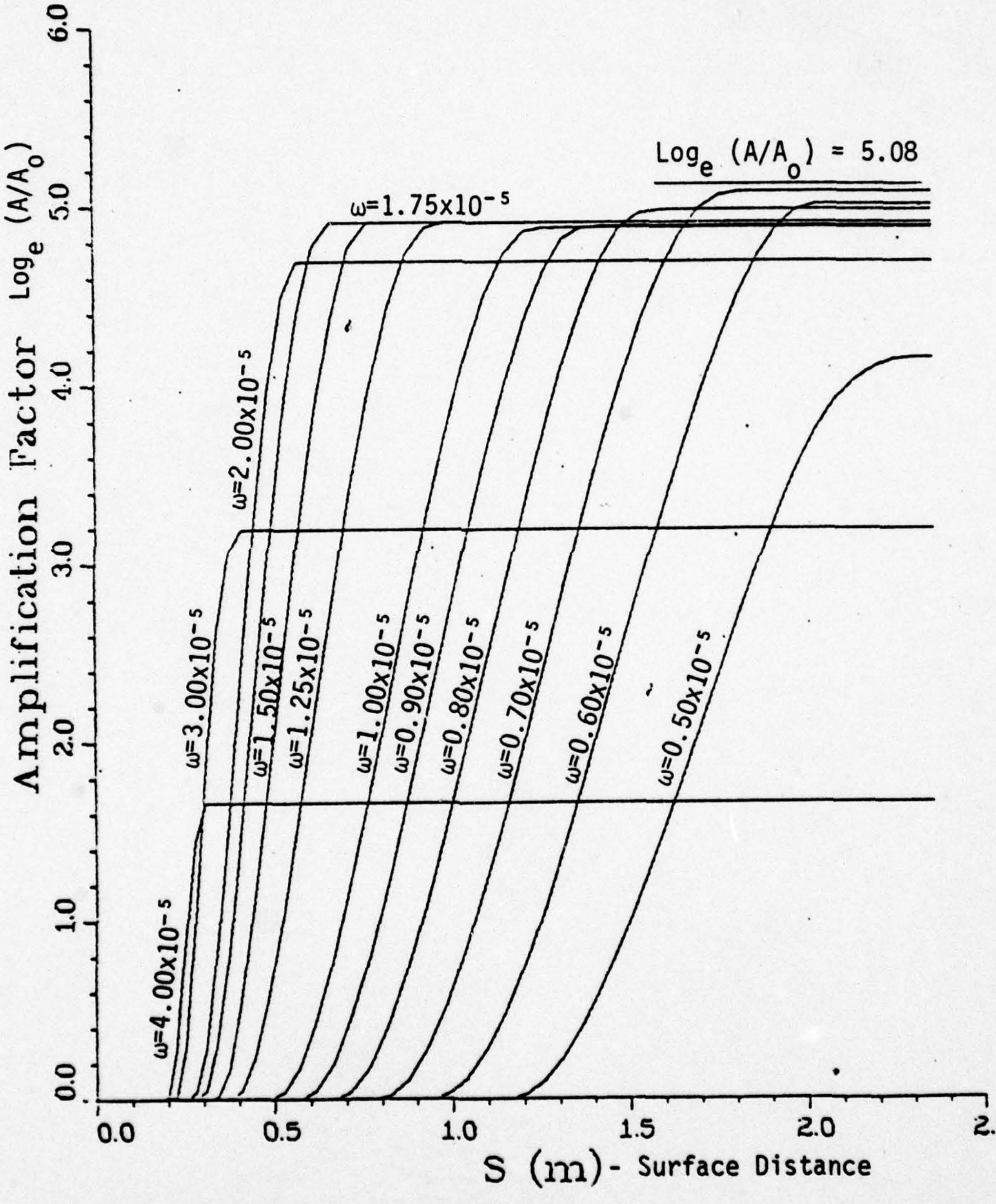



Figure 15. Spatial Amplification Factors in Boundary Layer, Method of Lowell & Reshotko - Parallel Flow - CWRU (A)

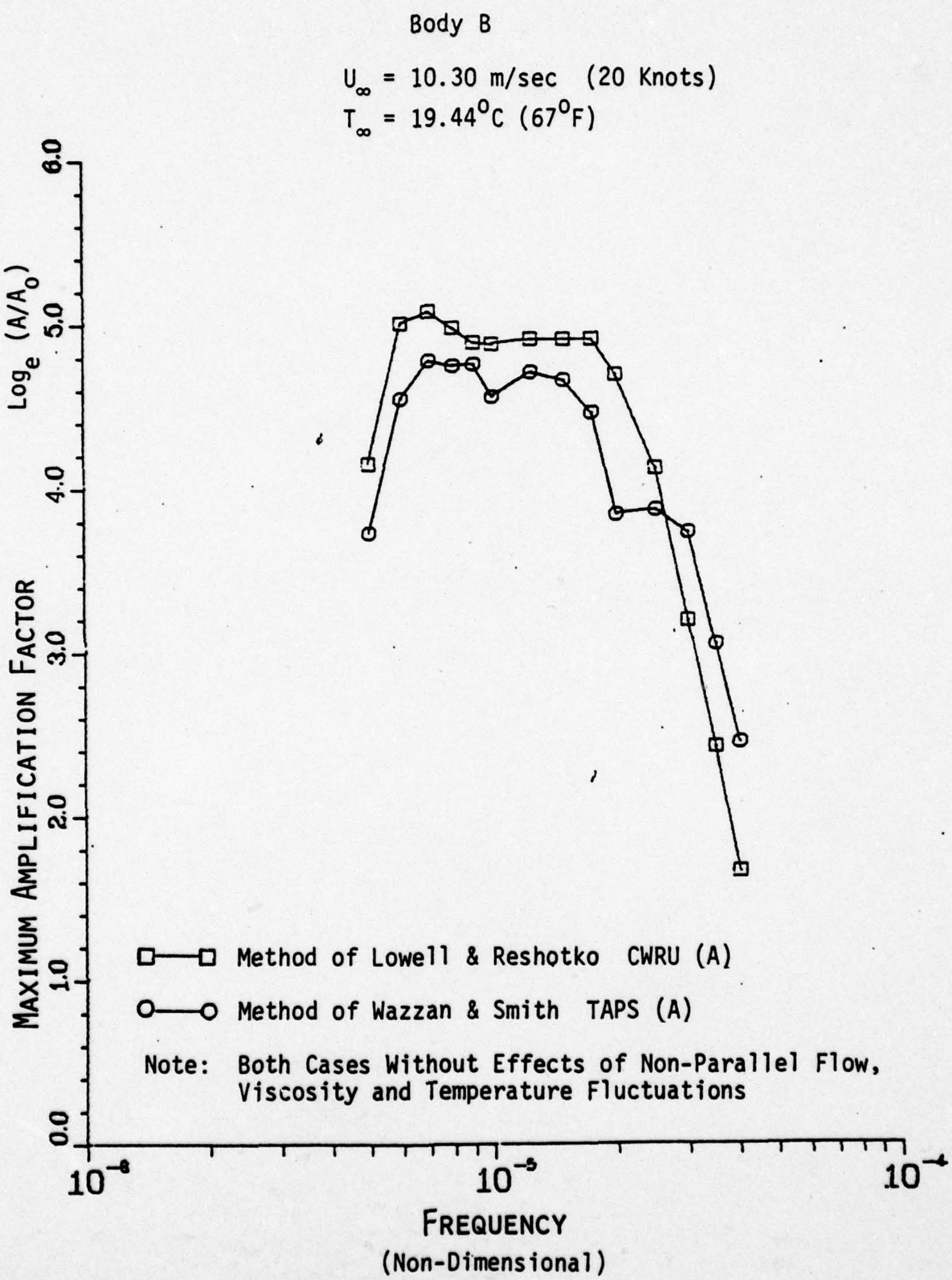



Figure 16. Effects of Numerical Solution on Linear Stability Analyses

### 4.3.3 Effects of Viscosity and Temperature Fluctuations

The resulting spatial amplification factors from the solution of equation (33) for the sample case are presented in Figure 17. A comparison of the computed stability results with [CWRU(B)] and without [CWRU(A)] effects of viscosity and temperature fluctuations terms is presented in Table 4 and Figure 18. Note that for this example, the inclusion of viscosity and temperature fluctuations reduces the amplification factors over the entire frequency range by an average of 19%.

Table 4. Effects of Viscosity and Temperature Fluctuations

| Non-Dimensional Frequency $\omega \times 10^5$ | Maximum Amplification Factors | | $\log_e (A/A_0)$ |
|---|---|---|---|
| | CWRU (A) | CWRU (B) | % Deviation from Mean Value |
| 4.00 | 1.66 | 1.09 | 21 |
| 3.50 | 2.42 | 1.72 | 17 |
| 3.00 | 3.19 | 2.33 | 16 |
| 2.50 | 4.12 | 3.12 | 14 |
| 2.00 | 4.69 | 4.01 | 8 |
| 1.75 | 4.91 | *4.03 | 10 |
| 1.50 | 4.91 | 3.68 | 14 |
| 1.25 | 4.91 | 2.54 | 32 |
| 1.00 | 4.88 | 2.49 | 32 |
| 0.90 | 4.89 | 1.49 | 53 |
| 0.80 | 4.98 | - | |
| 0.70 | *5.08 | - | |
| 0.60 | 5.01 | - | |
| 0.50 | 4.15 | - | |

* Highest Amplification Factor Ave. = 19%

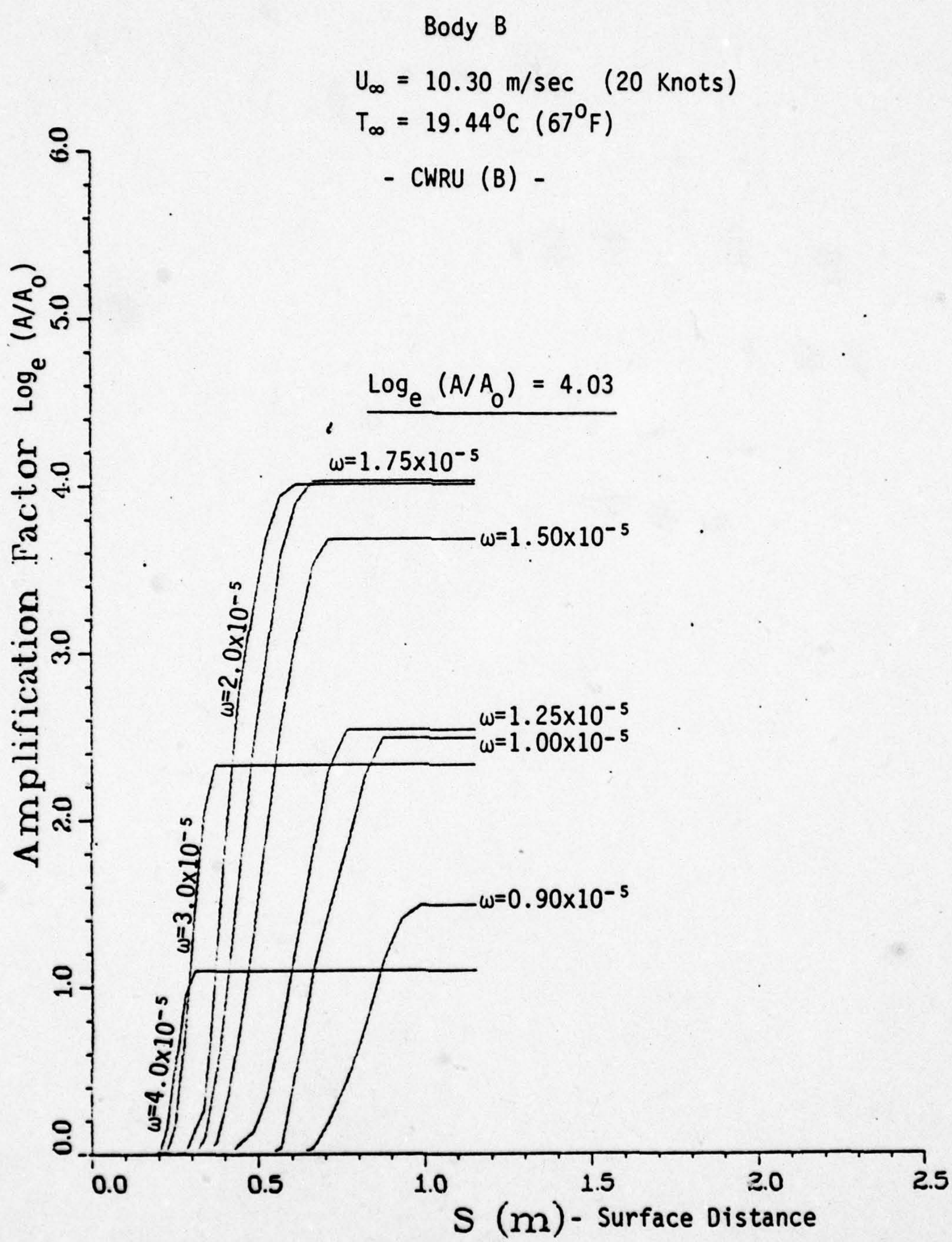



Figure 17. Spatial Amplification Factors in Boundary Layer, Method of Lowell & Reshotko - Including Effects of Viscosity and Temperature Fluctuations - CWRU (B)

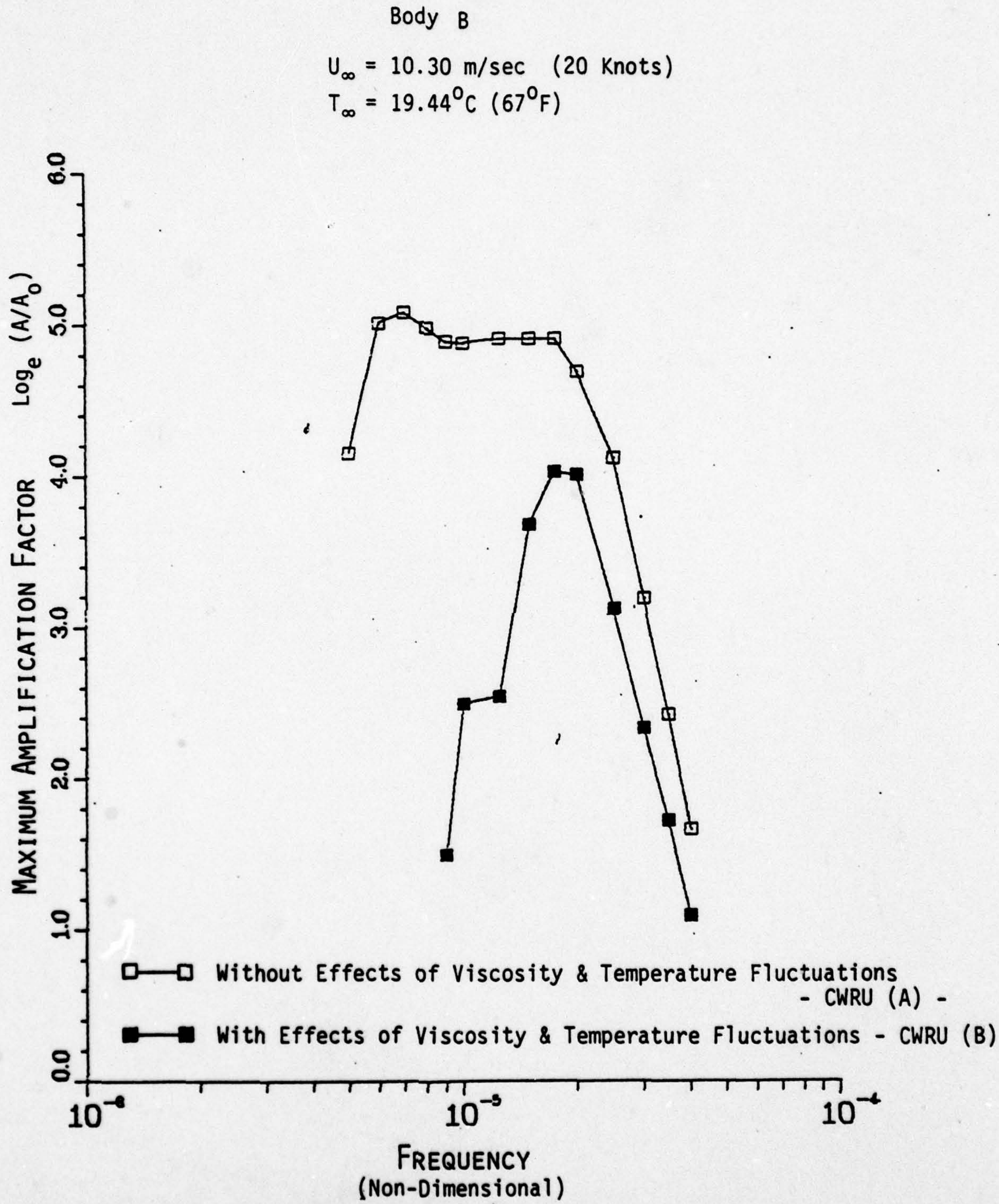



Figure 18. Effects of Viscosity and Temperature Fluctuations on Linear Stability Analyses

### 4.3.4 Effects of Fluid Properties

The boundary layer component of the TAPS Code has options for using air, pure water, or sea water as the working fluid (Gentry, 1976). For pure water, the fluid properties of Kaups and Smith (1967) are used. In the CWRU Code (Lowell & Reshotko (1974), in addition to the fluid properties of Kaups and Smith (1967), there is an option to use the fluid properties of Touloukian *et al.* (1970), Gildseth *et al.* (1972), Korosi *et al.* (1968), and Powell (1958).

A comparison between two different sets of fluid properties has been made by Lowell & Reshotko (1974). The resulting deviation of the form

$$\left(\left(\frac{Q}{Q_o}\right)_{K.S.} - \left(\frac{Q}{Q_o}\right)_{T.G.K.P.}\right) \times 100$$

versus Temperature $T(^{o}F)$ is presented in Figure 19, where Q is the fluid property at any temperature T and $Q_o$ is its value at $T = 32^{o}F$. At $T = 100^{o}F$ ($37.8^{o}C$), for example, the deviation varies from +0.5% to -0.7%.

Spatial amplification factors for the test example with effects of viscosity and temperature fluctuations but using the fluid properties of T.G.K.P. are presented in Figure 20. A comparison of the effects of fluid properties on stability results is presented in Table 5 and in Figure 21. The average percentage deviation from the mean value is only 3%. Hence, the effect is considered negligible.

Table 5. Effect of Fluid Properties

| Non-Dimensional Frequency $\omega \times 10^5$ | Maximum Amplification Factors CWRU (B) | CWRU (C) | $\log_e (A/A_o)$ % Deviation from Mean Value |
|---|---|---|---|
| 4.00 | 1.09 | 1.07 | 1 |
| 3.50 | 1.72 | 1.69 | 1 |
| 3.00 | 2.33 | 2.31 | 1 |
| 2.50 | 3.12 | 3.21 | 2 |
| 2.00 | 4.01 | 3.94 | 1 |
| 1.75 | *4.03 | *3.99 | 1 |
| 1.50 | 3.68 | 3.75 | 1 |
| 1.25 | 2.54 | 2.85 | 6 |
| 1.00 | 2.49 | 2.08 | 9 |
| 0.90 | 1.49 | 1.60 | 4 |

* Highest Amplification Factor Ave. = 3%

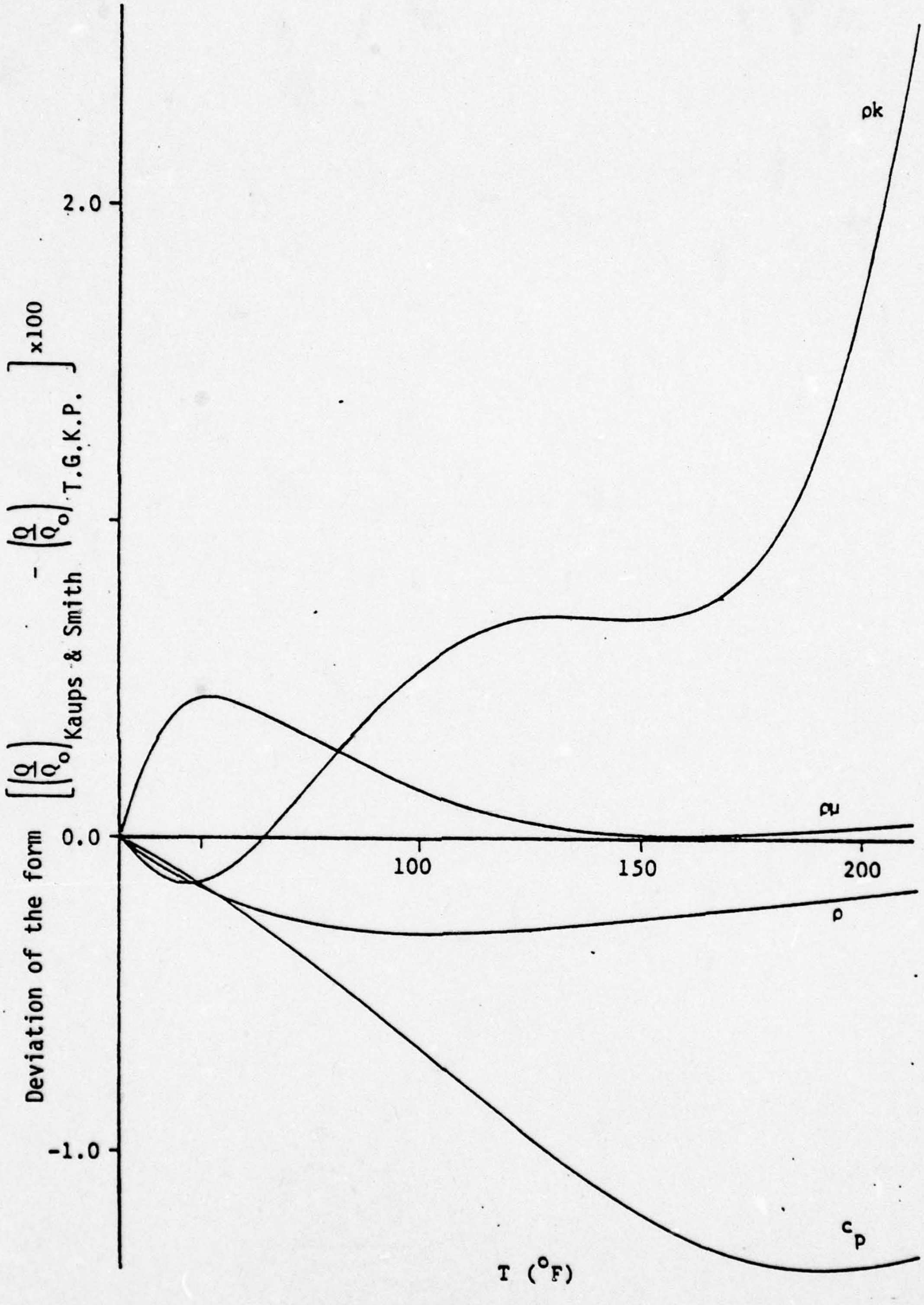



Figure 19. Deviation Between the Property-Temperature Variation of Kaups & Smith and that Specified by Touloukian, Gildseth, Korosi and Powell.
(Lowell & Reshotko 1974)

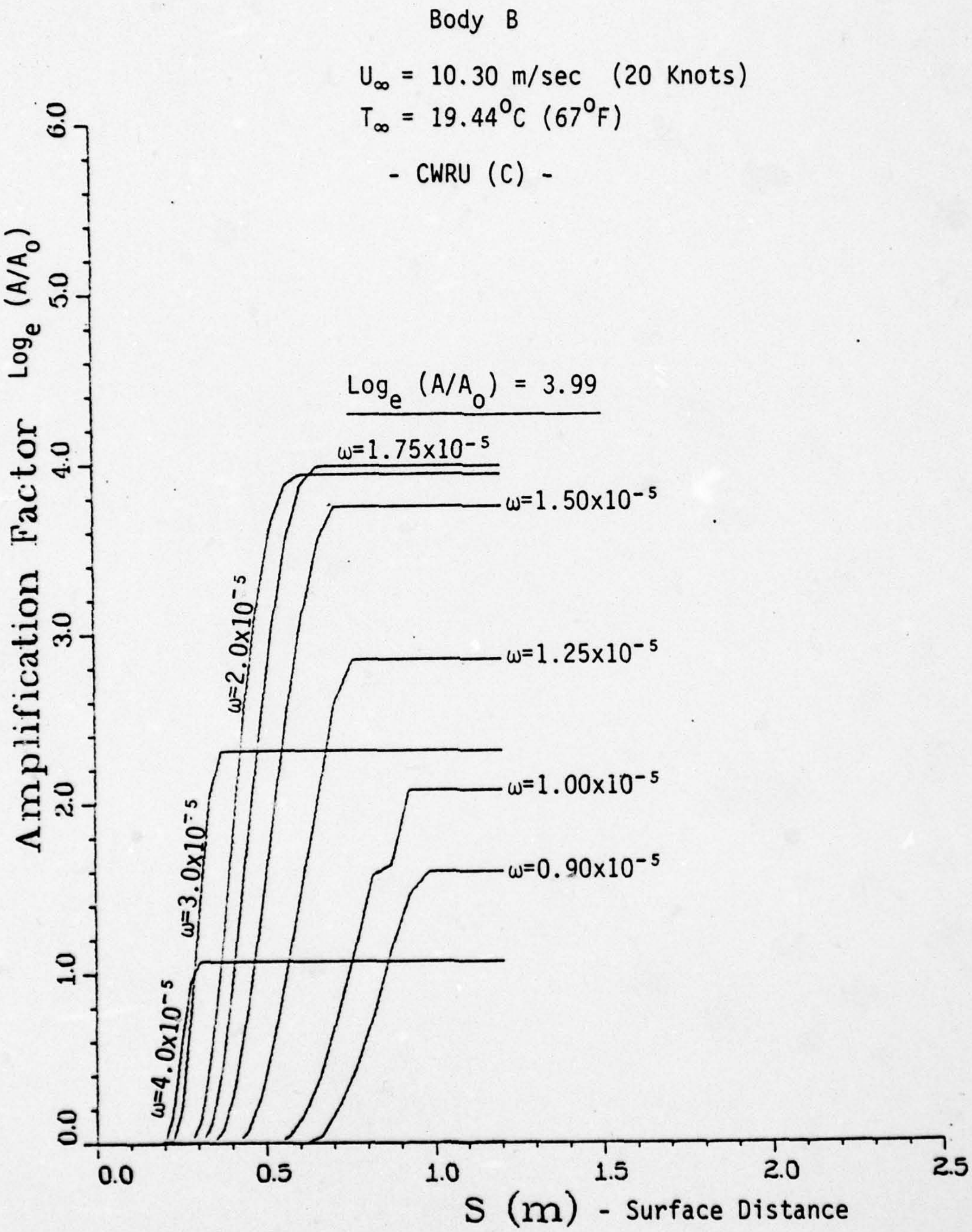



Figure 20. Spatial Amplification Factors in Boundary Layers, Method of Lowell & Reshotko - With Effects of Viscosity and Temperature Fluctuations, Fluid Properties of T.G.K.P. - CWRU (C)

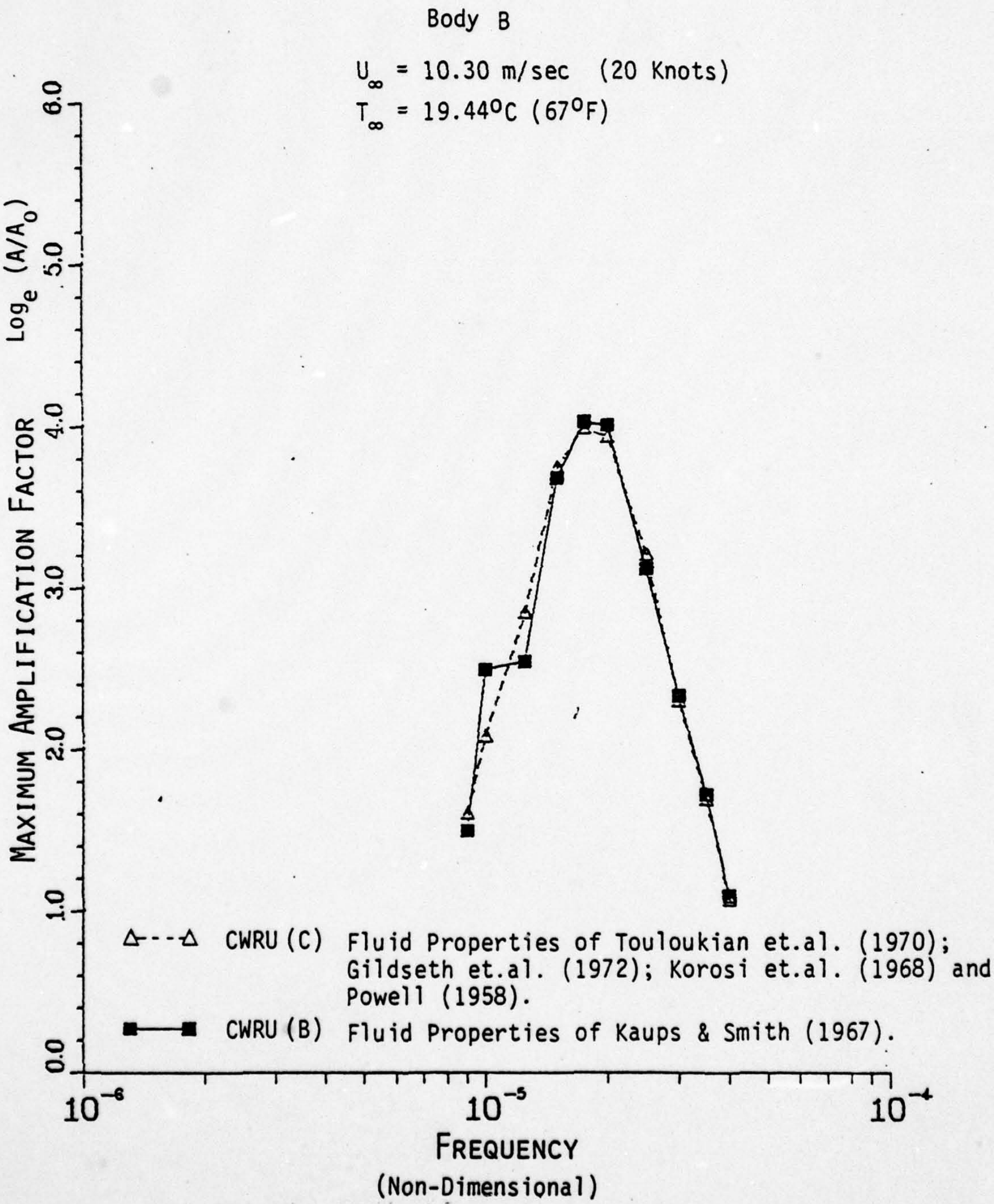



Figure 21. Effects of Fluid Properties on the Linear Stability Computation

## 4.4 Comparison of the Computational Efficiencies

The required computational time and cost for processing the stability calculation programs on a CDC 7600 Computer are listed as below:

CDC 7600 Computer

(10 Disturbance Frequencies)

| | SRU | Cost |
|---|---|---|
| TAPS (A) | 36.49 | $25.54 |
| TAPS (B) | 72.27* | $50.59 |
| CWRU (A) | 94.58* | $66.21 |

*The total SRU is estimated from two separate runs.

For the purpose of spatial amplification analysis, based on the current set-up of the computer programs, the TAPS Code (Wazzan & Smith) is less time consuming than the CWRU Code (Lowell & Reshotko). Furthermore, the required computational time for the partial non-parallel flow effect in the TAPS Code is almost double that of its parallel flow counterpart. Since the validity and utility of partial non-parallel flow effect remains to be justified, it is suggested that the parallel flow option for the stability analysis be used at the present time.

## 5. FURTHER DEVELOPMENT OF THE TAPS CODE, INCORPORATION OF THE ROUGHNESS MODEL

The Transition Analysis Program System (TAPS) is a rather large computer program designed to serve as an analytical tool for the prediction of boundary layer transition by the linear stability theory. The original TAPS program was assembled by Gentry et al (1976) at McDonnell Douglas Corporation. Further improvement of the operating efficiency of the program and the addition of James' Axisymmetric Inverse Potential Flow program have been accomplished by Lee et al (1978). The basic framework of the current version of the TAPS program is presented in Figure 22.

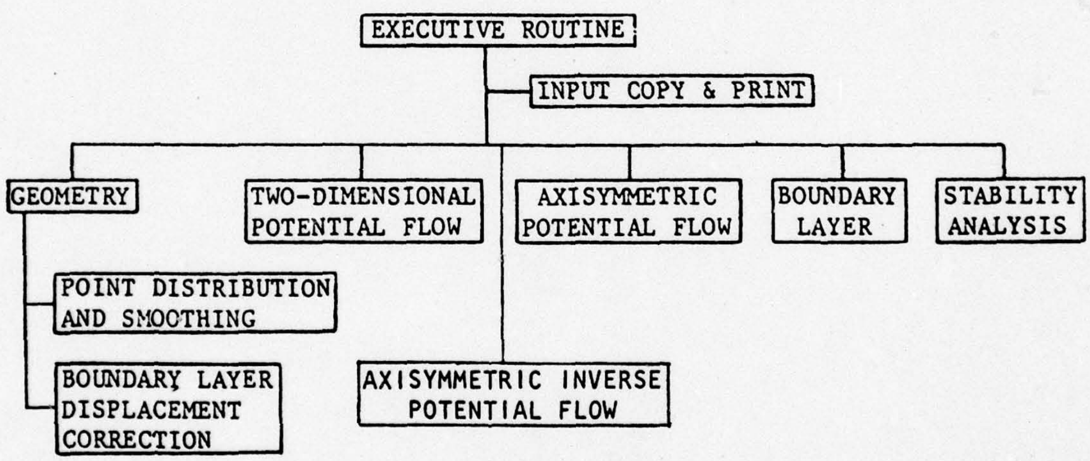



Figure 22. Basic Framework of the TAPS Program
(from Lee et al 1978)

For most engineering applications, only one or two of the components are used at a time. In this study, it was decided to extract three major components, Axisymmetric Potential Flow, Boundary Layer Flow and Stability Analysis from the TAPS Code.

Some simplifications have been made in output formats for these three components, but the logical procedures remain the same. In the event two or more of the components are used in the same run, a simple "Procedure File" can be used to direct the process.

At Dynamics Technology, under the sponsorship of DARPA, a roughness model was incorporated into the TAPS Code in 1977. A brief description of the approach is presented below:

## Incorporation of the Roughness Model into the TAPS Code

An analytical model for evaluating the effects of distributed surface roughness on transition characteristics was originally developed and discussed by Kosecoff, Ko and Merkle (1976). Further validation and incorporation of the model into the existing TAPS Code have been carried out by Merkle, Tzou and Kubota (1977). In their approach, the viscosity and thermal conductivity, $\mu$ and K are replaced by their rough-wall counterparts, $\mu_t$ and $K_t$ in the momentum and energy equations (Section 3, equations (26) and (27)). The definitions of $\mu_t$ and $K_t$ are

$$\mu_t = \mu + \rho\,\varepsilon_m \tag{35}$$

$$K_t = K + \rho\, c_p \varepsilon_H \tag{36}$$

where $\varepsilon_m$ and $\varepsilon_H$ are momentum and thermal diffusivities, respectively, and are calculated from the roughness model (Merkle, Tzou and Kubota, 1977).

The calculation procedure incorporating roughness follows the TAPS procedure in using a combination of Levy-Lees and Probstein-Elliott transformations (Gentry, 1976):

$$d\xi = \rho_e \mu_e u_e \left(\frac{r_o}{L}\right)^2 dx \tag{37}$$

$$d\eta = \frac{\rho u_e}{(2\xi)^{\frac{1}{2}}} \left(\frac{r}{L}\right) dy \tag{38}$$

$$r = r_o + y \cos \alpha \tag{39}$$

where $r_o$ is the body radius and $\alpha$ is the angle that the surface makes with the body axis, and L is the reference body length.

To simplify the procedure for incorporating the roughness model, the roughness height k is expressed in the transformed coordinate $\eta$. Thus, at a given streamwise position, the value of $\eta$ at the top of the roughness can be expressed by

$$\eta_k = \frac{u_e}{L(2\xi)^{\frac{1}{2}}} \int_0^k \rho r dy$$

$$\simeq \frac{\rho_e u_e}{L(2\xi)^{\frac{1}{2}}} \left[k\, r_o + \frac{k^2}{2} \cos \alpha\right]$$

$$\simeq \frac{\rho_e u_e}{L(2\xi)^{\frac{1}{2}}} \left[k\, r_o\right] \tag{40}$$

Once the roughness height is defined in the transformed coordinate system, its effect on the boundary layer characteristics can be computed at each streamwise location. The detailed procedure used for incorporating the roughness model into the TAPS Code can be found in the report by Merkle, Tzou and Kubota (1977). Figures 23 and 24 illustrate the effect of surface roughness on boundary-layer shape factor H for various smoothness conditions (k = 0, 8μm and 15μm) at the speeds of 20 knots and 40 knots. Based on the simple shape factor stability criterion, it clearly indicates that the surface roughness deteriorates the stability of boundary layers. The validity of this roughness model still requires more reliable experimental data. However, this model can be used for a first-order estimate of the effect of distributed surface roughness on the transition characteristics.

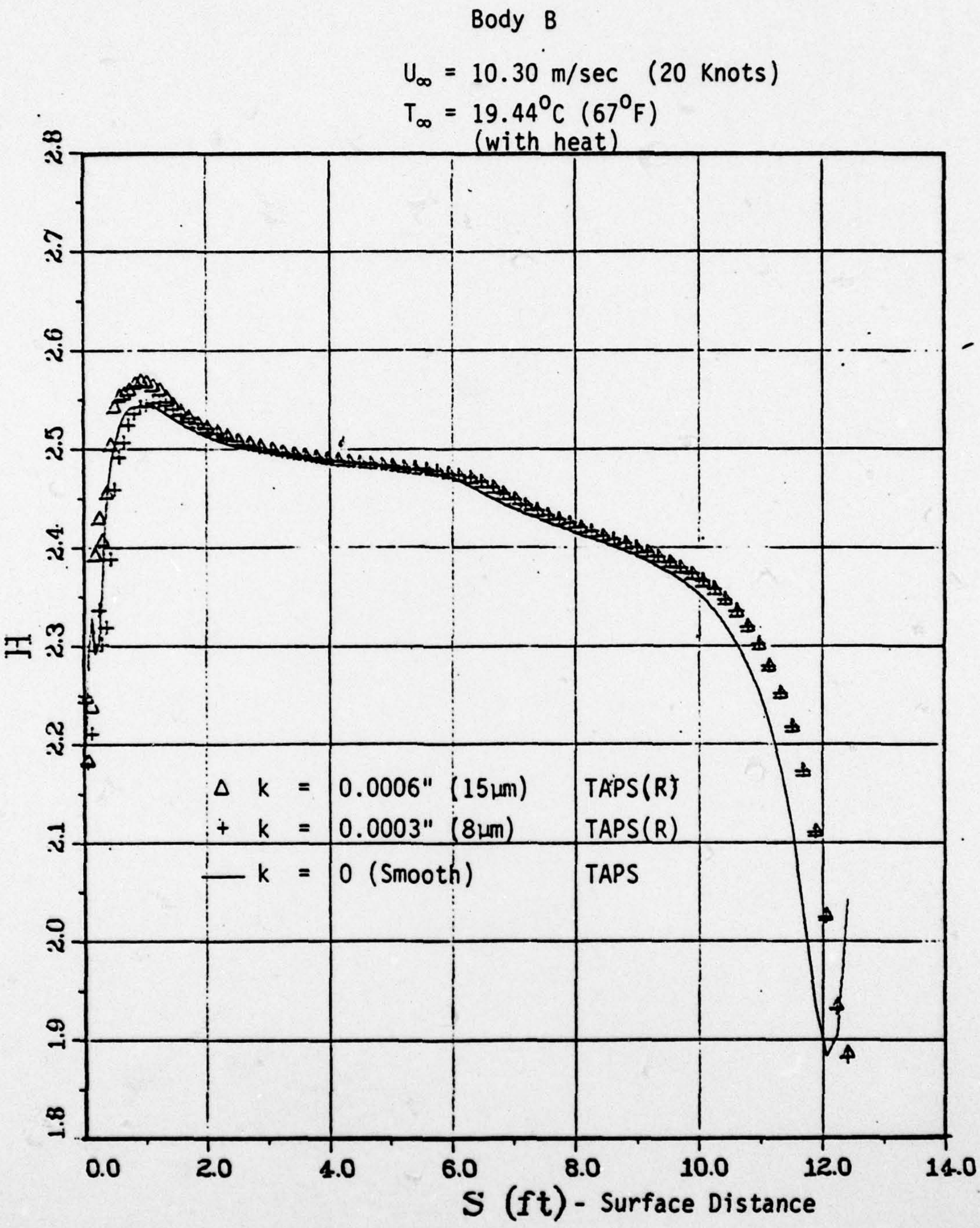



Figure 23. Effect of Surface Roughness on Boundary-Layer Shape Factor, H.

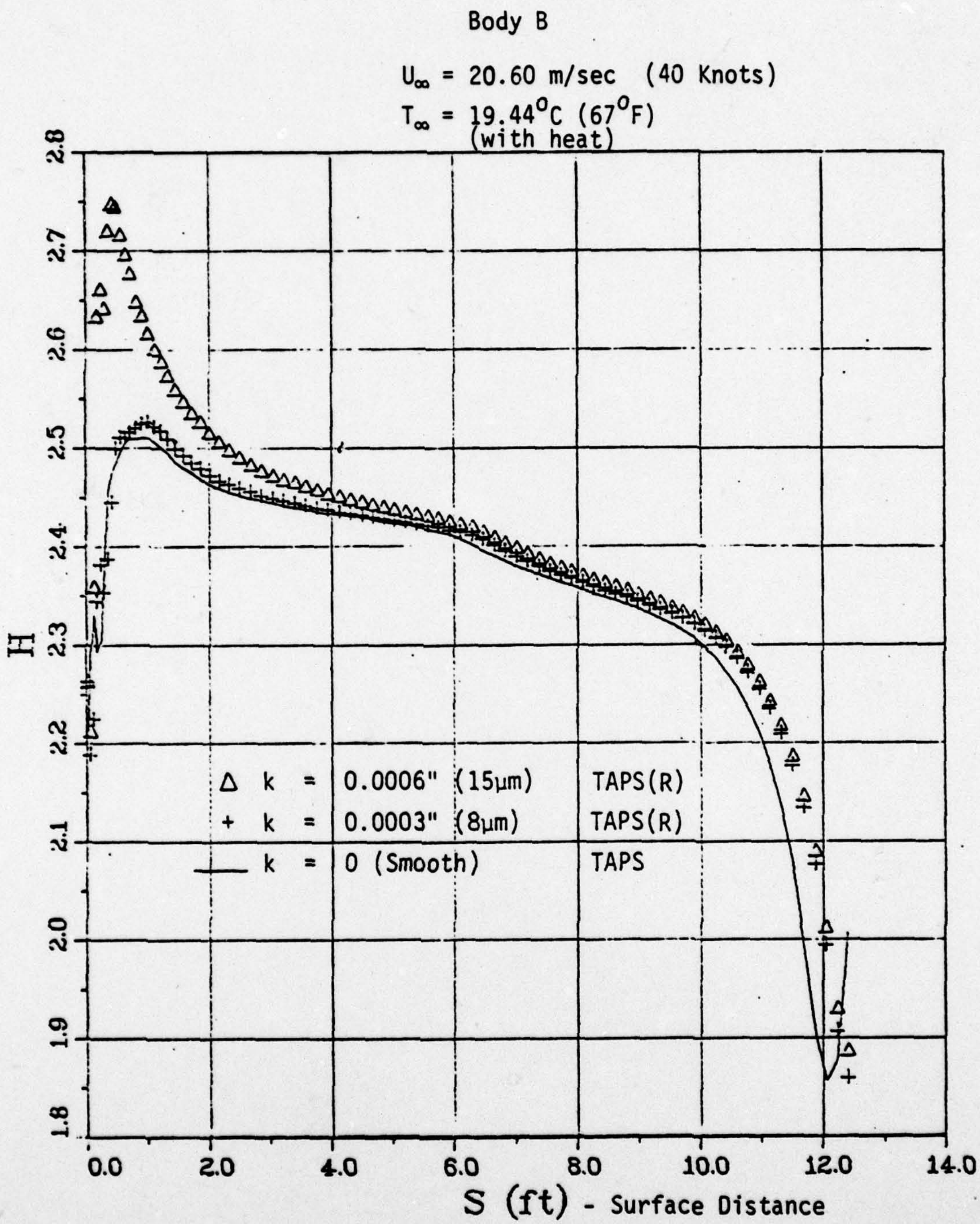



Figure 24. Effect of Surface Roughness on Boundary-Layer Shape Factor, H.

## 6. SUMMARY AND CONCLUSIONS

An assessment of the available analytical tools for analyzing boundary-layer transition on a heated axisymmetric body has been performed. The approach is focused on the evaluation of the validity and efficiency of the three major components in the TAPS Code, Axisymmetric Potential Flow, Boundary-Layer Flow, and Linear Stability Analysis. In addition, further development of the TAPS Code, through the incorporation of a roughness model, is also presented. Conclusions based on the numerical assessment are stated below.

### Axisymmetric Potential Flow

- The numerical approach of Landweber for solving the Fredholm integral equation of the first kind is relatively simple and less time consuming when compared with the approach of Hess and Smith for solving the Fredholm integral equation of the second kind. The results from both codes are sufficiently accurate for well-rounded bodies.

- For bodies with sudden changes in slope and curvature, or with local bumps, the approach of Hess and Smith can provide a better calculation than Landweber's approach.

### Boundary-Layer Flow

- The new code developed by Cebeci *et al.* (1978) appears to have merit for improving computational efficiency apparently because it solves the momentum and energy equations simultaneously rather than consecutively as in an earlier Cebeci-Smith (1974) code. Results from both codes are in good agreement.

## Stability Analysis

For the purpose of spatial amplification analysis, the stability component of the TAPS Code is less expensive than the code developed by Lowell and Reshotko. Effects of various factors on the stability results based on the limited number of cases studied are summarized as follows:

- The partial inclusion of non-parallel flow terms in the stability equation yields higher amplification factors than obtained from the parallel flow computation.
- For the same stability equation and fluid properties, the code developed by Lowell and Reshotko predicts a slightly higher amplification factor than the TAPS Code (difference in numerical method of solution).
- The stability analysis with the viscosity and temperature fluctuation terms included in the disturbance equations has predicted a slightly more stable flow than without the fluctuation terms.
- Variations of fluid properties from Kaups and Smith and from the fluid properties of Touloukian *et al.*, Gildseth *et al.*, Korosi *et al.*, and Powell, are very small in the range of practical interest. Their effects on the resulting stability analysis are insignificant.
- Since the validity of partial non-parallel flow effect in the TAPS Code is questionable and the computer code to account for the effect of full non-parallel flow with heat transfer is not available at the present time, the parallel flow option in the TAPS Code is recommended. But, bear in mind that some degree of conservatism should be taken when applying parallel flow stability results to growing boundary layers.

Roughness Model in the TAPS Code

- The applicability of the roughness model in the TAPS Code still requires more reliable experimental data. However, this model can be used for a first-order estimate of the effect of distributed roughness on the boundary-layer characteristics for designing a heated underwater vehicle.

## ACKNOWLEDGEMENTS

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